**Dawson College**: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 4** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** *(5 marks)* Solve for $X$ the following equation:

$$(AX+3I)^{-1}C = BA^{-1}C^T(BA^TC^{-1})^T$$

where $A = \begin{bmatrix} 13 & 5 \\ 5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 5 & 4 \\ 4 & 3 \end{bmatrix}$.

$$A^{-1} = \frac{1}{13(2)-5(5)}\begin{bmatrix} 2 & -5 \\ -5 & 13 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ -5 & 13 \end{bmatrix}$$

$$B^{-1} = \frac{1}{0(0)-(1)(-2)}\begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} \\ 1 & 0 \end{bmatrix}$$

$$(AX+3I)^{-1}C = BA^{-1}C^T(C^{-1})^T(A^T)^TB^T$$
$$(AX+3I)^{-1}CC^{-1} = BA^{-1}C^T(C^T)^{-1}AB^TC^{-1}$$
$$(AX+3I)^{-1} = BA^{-1}IAB^TC^{-1}$$
$$(AX+3I)^{-1} = BA^{-1}AB^TC^{-1}$$
$$((AX+3I)^{-1})^{-1} = (BIB^TC^{-1})^{-1}$$
$$AX+3I = (C^{-1})^{-1}(B^{-1})^TB^{-1}$$
$$AX = C(B^{-1})^TB^{-1} - 3I$$
$$A^{-1}AX = A^{-1}(C(B^{-1})^TB^{-1} - 3I)$$
$$X = A^{-1}C(B^{-1})^TB^{-1} - 3A^{-1}$$
$$X = \begin{bmatrix} 2 & -5 \\ -5 & 13 \end{bmatrix}\begin{bmatrix} 5 & 4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & 0 \end{bmatrix}\begin{bmatrix} 0 & -\frac{1}{2} \\ 1 & 0 \end{bmatrix} - 3\begin{bmatrix} 2 & -5 \\ -5 & 13 \end{bmatrix}$$
$$X = \begin{bmatrix} -10 & -7 \\ 27 & 19 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{4} \end{bmatrix} - \begin{bmatrix} 6 & -15 \\ -15 & 37 \end{bmatrix}$$
$$X = \begin{bmatrix} -10 & -7/4 \\ 27 & 19/4 \end{bmatrix} - \begin{bmatrix} 6 & -15 \\ -15 & 39 \end{bmatrix}$$
$$X = \begin{bmatrix} -16 & 53/4 \\ 42 & -137/4 \end{bmatrix}$$

**Question 2.** *(4 marks)* Assume that a square matrix $A$ satisfies $2A^2+5A-4I=0$. Show that $2A-I$ is invertible and find its inverse in terms of $A$ and $I$.

Lets show that $\exists B$ s.t. $(2A-I)B = I$
$B(2A-I) = I$

$$2A^2+5A-4I = 0$$
$$2A^2+5A-3I = I$$
$(2A-I)(A+3I) = I$ also $(A+3I)(2A-I) = I$

$\therefore$ $(2A-I)$ is invertible and its inverse is $B = (A+3I)$

**Question 3.** *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If $A$ and $B$ are square matrices of the same size then $(AB)^T = A^TB^T$.

False, Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $AB = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ $(AB)^T = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

but $A^T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $B^T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $A^TB^T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \neq (AB)^T$