Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 5** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Consider the matrices A, B, C, all square and of the same size. Assume that the linear systems $A\mathbf{x} = 0$ and $B\mathbf{x} = 0$ have only the trivial solution and that C is row equivalent to B. Prove that AC can be written as a product of elementary matrices.

Since $A\underline{x}=\underline{0}$ and $B\underline{x}=\underline{0}$ have only the trivial solution then A and B are invertible* by the equivalence theorem.

Since C is row equivalent to B $\exists E_i$ s.t. $C = E_k \cdots E_2 E_1 B$

$\therefore AC = AE_k \cdots E_2 E_1 B$ is invertible since it is a product of invertible matrices by * and since elementary matrices are invertible.

It follows by the equivalence theorem that AC can be written as a product of elementary matrices since AC is invertible.

Question 2.

a. *(4 marks)* Find the inverse of the matrix A using the inversion algorithm:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 5 \\ -2 & -2 & -11 \end{bmatrix}$$

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 5 & 0 & 1 & 0 \\ -2 & -2 & -11 & 0 & 0 & 1 \end{array}\right] \underset{2R_1+R_3\to R_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 5 & 0 & 1 & 0 \\ 0 & -2 & -9 & 2 & 0 & 1 \end{array}\right]$$

$$\underset{2R_2+R_3\to R_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 5 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 1 \end{array}\right]$$

$$\underset{\substack{-R_3+R_1\to R_1 \\ -5R_3+R_2\to R_2}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -2 & -1 \\ 0 & 1 & 0 & -10 & -9 & -5 \\ 0 & 0 & 1 & 2 & 2 & 1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & -2 & -1 \\ -10 & -9 & -5 \\ 2 & 2 & 1 \end{bmatrix}$$

b. *(2 marks)* Solve for x, y, z, where $\begin{bmatrix} x & y & z \end{bmatrix} A = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$ using the A^{-1} found in part a.

$$[x\ y\ z]AA^{-1} = [-1\ 0\ 1]A^{-1}$$

$$[x\ y\ z] = [-1\ 0\ 1]\begin{bmatrix} -1 & -2 & -1 \\ -10 & -9 & -5 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= [3\ 4\ 2]$$

c. *(2 marks)* Find two elementary matrices E_1 and E_2 which satisfy $E_2E_1A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$.

from part a)

E_1: $I \underset{2R_1+R_3\to R_3}{\sim} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = E_1$

E_2: $I \underset{2R_2+R_3\to R_3}{\sim} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} = E_2$