Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 6** name: ____________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If $(1,0,0,0,0)$ and $(2,0,0,0,0)$ are both solutions of a system of 13 linear equations then $(0,1,0,0,0)$ ____________ be another solution of the system.

Question 2.

a. *(3 marks)* Suppose A is an $m \times n$ matrix, B is an $n \times m$ matrix, and $BA = I_n$. Prove that if for some $\mathbf{b} \in \mathbb{R}^m$ the equation $A\mathbf{x} = \mathbf{b}$ has a solution, then that solution is unique.

b. *(3 marks)* Suppose A is an $m \times n$ matrix, C is an $n \times m$ matrix, and $AC = I_m$. Prove that the system $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b} \in \mathbb{R}^m$.

Question 3. *(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If A^2 is a symmetric matrix, then A is a symmetric matrix.

b. A matrix A is said to be *skew-symmetric* if $A^T = -A$. If A and B are skew-symmetric matrices such that $AB = -BA$ then AB is skew-symmetric.

Bonus Question. *(5 marks)* Let A and B be $m \times n$ and $n \times m$ matrices, respectively. If $m > n$, show that AB is singular.