Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 6** name: Y.Lamoutagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If $(1,0,0,0,0)$ and $(2,0,0,0,0)$ are both solutions of a system of 13 linear equations then $(0,1,0,0,0)$ Might be another solution of the system.

Question 2.

a. *(3 marks)* Suppose A is an $m\times n$ matrix, B is an $n\times m$ matrix, and $BA = I_n$. Prove that if for some $\mathbf{b}\in\mathbb{R}^m$ the equation $A\mathbf{x} = \mathbf{b}$ has a solution, then that solution is unique.

Suppose that the solution is not unique. That is. $\exists \underline{x}_1$ and $\underline{x}_2$ where $\underline{x}_1 \neq \underline{x}_2$

and $A\underline{x}_1 = \underline{b}$, $A\underline{x}_2 = \underline{b}$

We have $A\underline{x}_1 = A\underline{x}_2$

$BA\underline{x}_1 = BA\underline{x}_2$

$\underline{x}_1 = \underline{x}_2$ ↯

$\therefore$ the solution is unique.

b. *(3 marks)* Suppose A is an $m\times n$ matrix, C is an $n\times m$ matrix, and $AC = I_m$. Prove that the system $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}\in\mathbb{R}^m$.

Let $\underline{x} = C\underline{b}$, then $A\underline{x}$

$= AC\underline{b}$

$= I\underline{b}$

$= \underline{b}$

$\therefore$ $\underline{x}$ is a solution of $A\underline{x} = \underline{b}$

$\therefore$ $A\underline{x} = \underline{b}$ is consistent.

Question 3. *(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If A^2 is a symmetric matrix, then A is a symmetric matrix.

False, Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ then $A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore$ A^2 is symmetric but A is not symmetric

b. A matrix A is said to be *skew-symmetric* if $A^T = -A$. If A and B are skew-symmetric matrices such that $AB = -BA$ then AB is skew-symmetric. True

premise:

- $A^T = -A$
- $B^T = -B$
- $AB = -BA$

conclusion:

$(AB)^T = -(AB)$

LHS $= (AB)^T$

$= B^T A^T$

$= (-B)(-A)$ by premise

$= BA$

$= -AB$ by premise

Bonus Question. *(5 marks)* Let A and B be $m\times n$ and $n\times m$ matrices, respectively. If $m > n$, show that AB is singular.