Question 1. *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If the sum of the second and fourth row of a 6×6 matrix A is equal to the last row, then $\det(A)$ _____ be equal to zero.

Question 2. If A is an $n \times n$ matrix, the *characteristic polynomial* $c_A(x)$ of A is defined by $c_A(x) = \det(xI - A)$.

a. *(3 marks)* Find the eigenvalues λ of $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & 3 & -2 \end{bmatrix}$. That is, find the values of λ for which $c_A(\lambda) = 0$.

b. *(3 marks)* Show that if A is a square matrix then A and A^T have the same characteristic polynomial.

c. *(3 marks)* Show that for any 2×2 matrix A, $c_A(x) = x^2 - \text{trace}(A)x + \det A$.

Question 3. *(5 marks)* Given $\det A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2$; $B = \begin{bmatrix} 3g+a & 3h+b & 2 & 3i+c \\ d+2a & e+2b & 3 & f+2c \\ a & b & 4 & c \\ 0 & 0 & 5 & 0 \end{bmatrix}$. Find $\det B$.