Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 7** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If the sum of the second and fourth row of a 6×6 matrix A is equal to the last row, then $\det(A)$ must be equal to zero.

Question 2. If A is an $n\times n$ matrix, the *characteristic polynomial* $c_A(x)$ of A is defined by $c_A(x)=\det(xI-A)$.

a. *(3 marks)* Find the eigenvalues λ of $A=\begin{bmatrix}2&0&0\\1&2&-1\\1&3&-2\end{bmatrix}$. That is, find the values of λ for which $c_A(\lambda)=0$.

$$\begin{aligned}
0 &= c_A(\lambda)\\
&= \det(\lambda I - A)\\
&= \det\left(\begin{bmatrix}\lambda-2 & 0 & 0\\ -1 & \lambda-2 & 1\\ -1 & -3 & \lambda+2\end{bmatrix}\right)\\
&= a_{11}C_{11} + \underbrace{a_{12}}_{0}C_{12} + \underbrace{a_{13}}_{0}C_{13}\\
&= (\lambda-2)\begin{vmatrix}\lambda-2 & 1\\ -3 & \lambda+2\end{vmatrix}\\
&= (\lambda-2)\left[(\lambda-2)(\lambda+2)+3\right]\\
&= (\lambda-2)\left[\lambda^2-4+3\right]\\
&= (\lambda-2)(\lambda^2-1) = (\lambda-2)(\lambda+1)(\lambda-1) \quad \therefore \lambda = 2,-1,1
\end{aligned}$$

b. *(3 marks)* Show that if A is a square matrix then A and A^T have the same characteristic polynomial.

$$\begin{aligned}
c_{A^T}(x) &= \det(xI - A^T)\\
&= \det(xI^T - A^T)\\
&= \det((xI)^T - A^T)\\
&= \det((xI-A)^T)\\
&= \det(xI-A)\\
&= c_A(x)
\end{aligned}$$

c. *(3 marks)* Show that for any 2×2 matrix A, $c_A(x)=x^2-\operatorname{trace}(A)x+\det A$.

Let $A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$, $\operatorname{trace}(A)=a+d$, $\det(A)=ad-bc$

$$\begin{aligned}
c_A(x) &= \det(xI-A)\\
&= \det\left(\begin{bmatrix}x-a & -b\\ -c & x-d\end{bmatrix}\right)\\
&= (x-a)(x-d)-cb\\
&= x^2-ax-dx+ad-cb\\
&= x^2-(a+d)x+ad-cb\\
&= x^2-\operatorname{trace}(A)x+\det A
\end{aligned}$$

Question 3. *(5 marks)* Given $\det A=\begin{vmatrix}a&b&c\\d&e&f\\g&h&i\end{vmatrix}=2$; $B=\begin{bmatrix}3g+a & 3h+b & 2 & 3i+c\\ d+2a & e+2b & 3 & f+2c\\ a & b & 4 & c\\ 0&0&5&0\end{bmatrix}$. Find $\det B$.

$$\begin{aligned}
|B| &= b_{41}C_{41}+b_{42}C_{42}+b_{43}C_{43}+b_{44}C_{44}\\
&= 5(-1)^{4+3}\begin{vmatrix}3g+a & 3h+b & 3i+c\\ d+2a & e+2b & f+2c\\ a & b & c\end{vmatrix}\\
&= -5 \quad \begin{matrix}-R_3+R_1\to R_1\\ -2R_3+R_2\to R_2\end{matrix}\begin{vmatrix}3g & 3h & 3i\\ d & e & f\\ a & b & c\end{vmatrix}\\
&= -5 \quad \tfrac{1}{3}R_1\to R_1 \quad (3)\begin{vmatrix}g & h & i\\ d & e & f\\ a & b & c\end{vmatrix} = -15 \quad R_1 \leftrightarrow R_3 \quad (-1)\begin{vmatrix}a & b & c\\ d & e & f\\ g & h & i\end{vmatrix} = 15(2) = 30
\end{aligned}$$