Question 1. *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

   a. If A is a square matrix and $\det(A + A^T) = 0$ then A _____ be singular.

Question 2. If A is an $n \times n$ matrix, the *characteristic polynomial* $c_A(x)$ of A is defined by $c_A(x) = \det(xI - A)$.

   a. *(3 marks)* Show that if A is an $n \times n$ matrix then $c_{A^2}(x^2) = (-1)^n c_A(x) c_A(-x)$.

Question 3. *(3 marks)* Given $\det A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2, B = \begin{bmatrix} 3g+a & 3h+b & 2 & 3i+c \\ d+2a & e+2b & 3 & f+2c \\ a & b & 4 & c \\ 0 & 0 & 5 & 0 \end{bmatrix}$ and $\det B = 30$. Find $\det\left(5A^4(A^{-1})^T B^{-3} \mathrm{adj}(A)\right)$.

Question 3. *(3 marks)* If A is an an $n \times n$ matrix where $\det(A) = x \neq 0$ then determine for which value(s) of x, if any, the matrix $A + \mathrm{adj}(A^{-1})$ is invertible.

Question 4. *(1 mark)* Correctly and precisely state Cramer's Rule.

Bonus Question. *(5 marks)* Prove Cramer's Rule.