Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 9** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

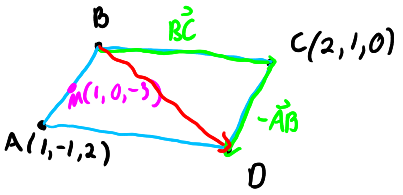
a. Two equivalent vectors might have the same initial point.

b. If $\mathbf{u}\cdot\mathbf{v}=\mathbf{u}\cdot\mathbf{w}$, then $\mathbf{v}$ might be equal to $\mathbf{w}$.

c. Let $\mathbf{w}$ be orthogonal to both $\mathbf{u}$ and $\mathbf{v}$. Then $\mathbf{w}$ must be orthogonal to $\mathbf{u}+\mathbf{v}$.

d. Let $\mathbf{u}$ be parallel to $\mathbf{x}$, and let $\mathbf{v}$ be parallel to $\mathbf{y}$. Then $\mathbf{u}+\mathbf{v}$ might be parallel to $\mathbf{x}+\mathbf{y}$.

Question 2. *(4 marks)* A parallelogram has sides AB, BC, CD, and DA. Given $A(1,-1,2)$, $C(2,1,0)$, and the midpoint $M(1,0,-3)$ of AB, find $\vec{BD}$.



$\frac{1}{2}\vec{AB}=\vec{AM}$

$\frac{1}{2}\vec{AB}=\vec{OM}-\vec{OA}$

$\vec{AB}=2[(1,0,-3)-(1,-1,2)]$

$\vec{AB}=2(0,1,-5)$

$\boxed{\vec{AB}=(0,2,-10)}$

$\vec{OB}-\vec{OA}=(0,2,-10)$

$\vec{OB}=(0,2,-10)+\vec{OA}$

$\vec{OB}=(0,2,-10)+(1,-1,2)$

$\vec{OB}=(1,1,-8)$

$\therefore B=(1,1,-8)$

$\vec{BC}=\vec{OC}-\vec{OB}$

$=(2,1,0)-(1,1,-8)$

$=(1,0,8)$

$\vec{BD}=\vec{BC}+(-\vec{AB})$

$=(1,0,8)+(0,-2,10)$

$=(1,-2,18)$

Question 3. *(4 marks)* Let $\mathbf{u}$ be a unit vector, and let $\mathbf{v}$ be a vector such that $||\mathbf{v}||=3$, and $||2\mathbf{u}-\mathbf{v}||=\sqrt{19}$. Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

$\|\underline{u}\|=1$

$\sqrt{19}=\|2\underline{u}-\underline{v}\|$

$(\sqrt{19})^2=\|2\underline{u}-\underline{v}\|^2$

$19=(2\underline{u}-\underline{v})\cdot(2\underline{u}-\underline{v})$

$19=(2\underline{u})\cdot(2\underline{u})-(2\underline{u})\cdot\underline{v}-\underline{v}\cdot(2\underline{u})+\underline{v}\cdot\underline{v}$

$19=4\underline{u}\cdot\underline{u}-2\underline{u}\cdot\underline{v}-2\underline{v}\cdot\underline{u}+\|\underline{v}\|^2$

$19=4\|\underline{u}\|^2-2\underline{u}\cdot\underline{v}-2\underline{u}\cdot\underline{v}+3^2$

$19=4(1)^2-4\underline{u}\cdot\underline{v}+9$

$6=-4\underline{u}\cdot\underline{v}$

$-\frac{3}{2}=\underline{u}\cdot\underline{v}$

$-\frac{3}{2}=\|\underline{u}\|\|\underline{v}\|\cos\theta$

$-\frac{3}{2}=(1)(3)\cos\theta$

$-\frac{1}{2}=\cos\theta$

$\theta=\frac{2\pi}{3}$