Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S3: Winter 2023: **Quiz 9** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Find the exact length of the curve $y = \ln(\cos x)$ on $[0, \pi/3]$.

$$S = \int_0^{\pi/3} \sqrt{1 + (y')^2}\, dx \qquad y' = \frac{1}{\cos x} \cdot -\sin x = -\tan x$$

$$= \int_0^{\pi/3} \sqrt{1 + (-\tan x)^2}\, dx$$

$$= \int_0^{\pi/3} \sqrt{1 + \tan^2 x}\, dx$$

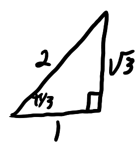
$$= \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx$$

$$= \int_0^{\pi/3} |\sec x|\, dx$$

$$= \int_0^{\pi/3} \sec x\, dx \qquad \text{since } \sec x > 0 \text{ on } [0, \tfrac{\pi}{3}]$$

$$= \Big[\ln|\sec x + \tan x|\Big]_0^{\pi/3}$$

$$= \ln|\sec \tfrac{\pi}{3} + \tan \tfrac{\pi}{3}| - \ln|\sec 0 + \tan 0|$$



$$= \ln|2 + \sqrt{3}| - \ln 1$$

$$= \ln(2 + \sqrt{3})$$

Question 2. *(5 marks)* Sketch and find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1,1)$, and the x-axis.

tangent of $y = x^2$ at $(1,1)$:

$y' = 2x$

$m = 2(1) = 2$ ∴ slope of tangent at $x = 1$ is 2

∴ $y = mx + b$

$y = 2x + b$

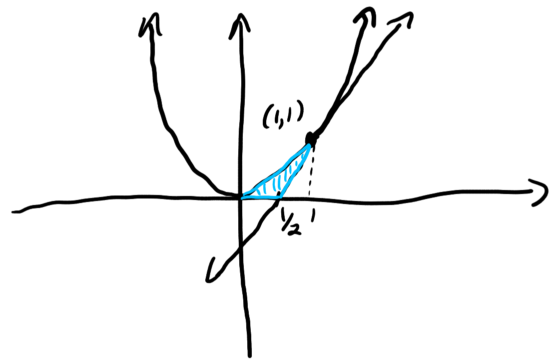
sub $(1,1)$

$1 = 2(1) + b$

$b = -1$

∴ $y = 2x - 1$

The x-int. of the tangent is $x = \frac{1}{2}$



$$\text{Area} = \int_0^{1/2} x^2 - 0\,dx + \int_{1/2}^{1} x^2 - (2x-1)\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^{1/2} + \left[\frac{x^3}{3} - \frac{2x^2}{2} + x\right]_{1/2}^{1}$$

$$= \cancel{\frac{(1/2)^3}{3}} + \left[\frac{1^3}{3} - 1^2 + 1\right] - \left[\cancel{\frac{(1/2)^3}{3}} - (1/2)^2 + 1/2\right]$$

$$= 1/3 + 1/4 - 1/2$$

$$= \frac{4+3-6}{12}$$

$$= \frac{1}{12}$$