Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2024: **Quiz 10** name: Y. Lamontagne

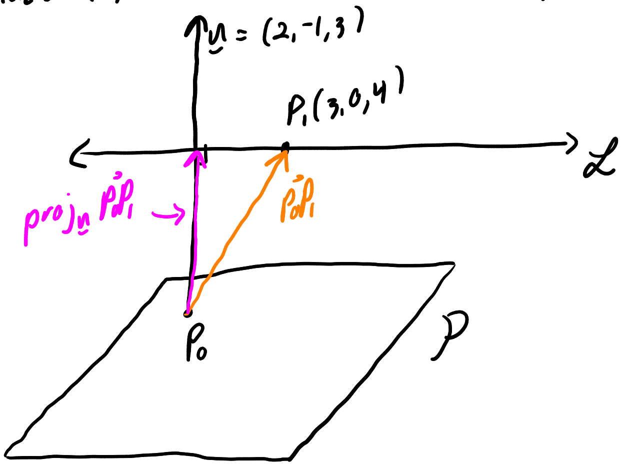
Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Given the plane $\mathscr{P}: 2x_1 - x_2 + 3x_3 = 5$, and the line $\mathscr{L}: \mathbf{x} = (3,0,4) + t(-1,1,1)$ where $t \in \mathbb{R}$, determine if the line is parallel to the plane, orthogonal to the plane, or neither parallel nor orthogonal. If possible find the intersection between $\mathscr{P}$ and $\mathscr{L}$, justify. Also if possible find the distance between $\mathscr{P}$ and $\mathscr{L}$, justify.

$\underline{n} \not\parallel \underline{d}$ since $\nexists k$ s.t. $\underline{n} = k\underline{d}$ $\therefore \mathscr{P} \not\perp \mathscr{L}$

$\underline{n} \cdot \underline{d} = (2,-1,3) \cdot (-1,1,1) = -2 - 1 + 3 = 0$ $\therefore \mathscr{P} \parallel \mathscr{L}$

Since $(3,0,4)$ does not lie on the plane, there is no intersection between $\mathscr{L}$ and $\mathscr{P}$.



A point on the plane is $P_0(1,0,1)$ since it satisfies the eqn of the plane

$\vec{P_0P_1} = \vec{OP_1} - \vec{OP_0} = (3,0,4) - (1,0,1) = (2,0,3)$

$\text{proj}_{\underline{n}} \vec{P_0P_1} = \dfrac{\vec{P_0P_1} \cdot \underline{n}}{\underline{n} \cdot \underline{n}} \underline{n}$

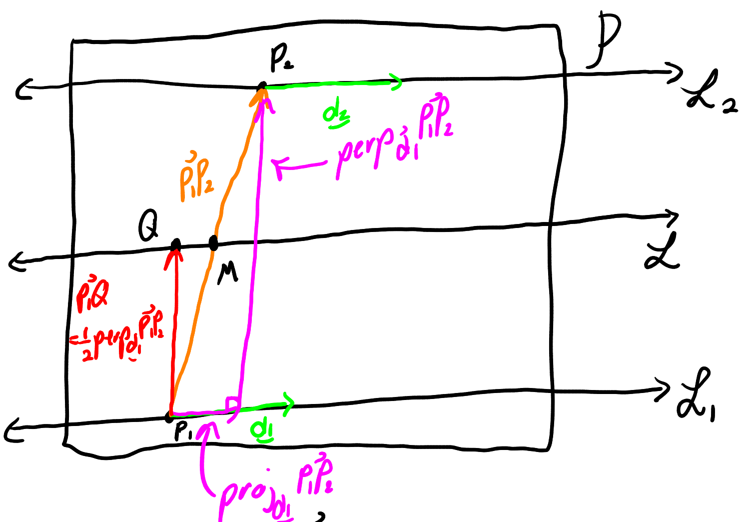
$= \dfrac{(2,0,3) \cdot (2,-1,3)}{(2,-1,3) \cdot (2,-1,3)} (2,-1,3)$

$= \dfrac{4+9}{4+1+9} (2,-1,3) = \dfrac{13}{14}(2,-1,3)$

distance $= \| \text{proj}_{\underline{n}} \vec{P_0P} \|$

$= \| \frac{13}{14}(2,-1,3) \| = \frac{13}{14} \| (2,-1,3) \| = \frac{13}{14}\sqrt{4+1+9} = \frac{13\sqrt{14}}{14}$

Question 2. *(5 marks)* Given that $\mathscr{L}_1: \mathbf{x} = (1,0,2) + t(-1,3,2)$, $\mathscr{L}_2: \mathbf{x} = (1,1,-1) + t(-1,3,2)$ where $t \in \mathbb{R}$ that lie on the same plane $\mathscr{P}$. Find the equation of the line $\mathscr{L}$ which lies on $\mathscr{P}$ and is equidistant from $\mathscr{L}_1$ and $\mathscr{L}_2$.



Lets find a point Q on $\mathscr{L}$.

$\vec{P_1P_2} = \vec{OP_2} - \vec{OP_1} = (1,1,-1) - (1,0,2) = (0,1,-3)$

$\text{perp}_{\underline{d_1}} \vec{P_1P_2} = \vec{P_1P_2} - \text{proj}_{\underline{d_1}} \vec{P_1P_2}$

$= (0,1,-3) - \dfrac{\vec{P_1P_2} \cdot \underline{d_1}}{\underline{d_1} \cdot \underline{d_1}} \underline{d_1}$

$= (0,1,-3) - \dfrac{(0,1,-3) \cdot (-1,3,2)}{(-1,3,2) \cdot (-1,3,2)} (-1,3,2)$

$= (0,1,-3) - \dfrac{-3}{1+9+4}(-1,3,2)$

$= (0,1,-3) + \dfrac{3}{14}(-1,3,2) = \left(\dfrac{-3}{14}, \dfrac{23}{14}, \dfrac{-36}{14}\right)$

$\vec{P_1Q} = \frac{1}{2} \text{perp}_{d_1} \vec{P_1P_2}$

$\vec{OQ} - \vec{OP_1} = \frac{1}{2}\left(\frac{-3}{14}, \frac{23}{14}, \frac{-18}{14}\right)$

$\vec{OQ} = (1,0,2) + \frac{1}{28}(-3,23,-36) = \left(\frac{25}{28}, \frac{23}{28}, \frac{20}{28}\right)$ $\therefore \mathscr{L}: \underline{x} = \left(\frac{25}{28}, \frac{23}{28}, \frac{20}{28}\right) + t(-1,3,-2)$ where $t \in \mathbb{R}$

Bonus Question. *(3 marks)* Prove the Cauchy-Schwartz Inequality by using the squared norm of $||\vec{u}||\vec{v} - ||\vec{v}||\vec{u}$ and $||\vec{u}||\vec{v} + ||\vec{v}||\vec{u}$.

Or $M = \frac{1}{2}(P_1 + P_2) = \frac{1}{2}((1,0,2) + (1,1,-1)) = (1, \frac{1}{2}, \frac{1}{2})$ lies on the line

$\mathscr{L}: \underline{x} = (1, \frac{1}{2}, \frac{1}{2}) + t(-1,3,2)$ where $t \in \mathbb{R}$