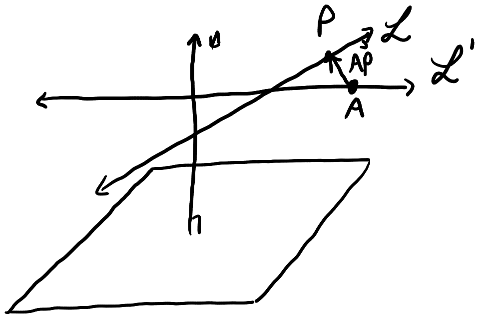
Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2024: **Quiz 11** name: *Y. Lamontagne*

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Given the line $\mathscr{L} : (x,\ y,\ z) = (2,\ 2,\ 3) + t(1,\ -1,\ -3)$ where $t \in \mathbb{R}$, the plane $\mathscr{P} : 3x - 2y + 2z = 7$ and the point $A(1,1,1)$.

a. *(4 marks)* Find parametric equations of the line which contains A, intersects $\mathscr{L}$ and which is parallel to $\mathscr{P}$.



$\vec{AP} = \vec{OP} - \vec{OA} = (2+t,\ 2-t,\ 3-3t) - (1,1,1) = (1+t,\ 1-t,\ 2-3t)$

For $\mathscr{L}' \parallel \mathscr{P}$ we need

$\vec{AP} \cdot \underline{n} = 0$

$0 = (1+t, 1-t, 2-3t) \cdot (3,-2,2)$

$0 = 3(1+t) - 2(1-t) + 2(2-3t)$

$0 = 3 + 3t - 2 + 2t + 4 - 6t$

$0 = 5 - t$

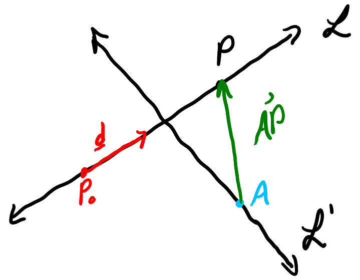
$t = 5$

So $\underline{d} = \vec{AP}$ when $t = 5$

$= (1+5,\ 1-5,\ 2-3(5)) = (6, -4, -13)$

$\therefore \mathscr{L}' : \underline{x} = (1,1,1) + t(6,-4,-13)$ where $t \in \mathbb{R}$.

b. *(4 marks)* Find parametric equations of the line which contains A and which intersects $\mathscr{L}$ at a right angle.



For $\mathscr{L} \perp \mathscr{L}'$ we need

$\vec{AP} \cdot \underline{d} = 0$

$0 = (1+t, 1-t, 2-3t) \cdot (1,-1,-3)$

$0 = 1(1+t) - (1-t) - 3(2-3t)$

$0 = 1 + t - 1 + t - 6 + 9t$

$0 = -6 + 11t$

$t = \frac{6}{11}$

So $\underline{d} = \vec{AP}$ when $t =$

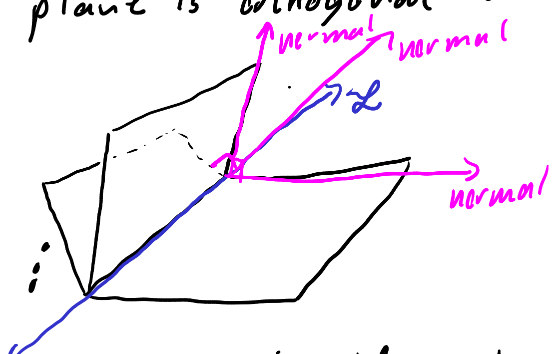
$= (1+\frac{6}{11},\ 1-\frac{6}{11},\ 2-3(\frac{6}{11}))$

$= (\frac{17}{11}, \frac{5}{11}, \frac{4}{11})$

$\therefore \mathscr{L}' : \underline{x} = (1,1,1) + t(17,5,4)$ where $t \in \mathbb{R}$

Question 2. *(4 marks)* If $(3,\ -2,\ 1)$ is a particular solution of $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x} = t\overbrace{(1,\ 3,\ -2)}^{d_1}$ where $t \in \mathbb{R}$ is the solution set of $A\mathbf{x} = \mathbf{0}$. Give a geometric interpretation of the system $A\mathbf{x} = \mathbf{0}$ and its solution set. Determine whether $\underbrace{\mathbf{x} = (4,\ 1,\ -1) + t(-2,\ -6,\ 4)}_{\mathscr{L}'}$ where $t \in \mathbb{R}$ is the solution set of $A\mathbf{x} = \mathbf{b}$, justify.

The system $A\underline{x} = \underline{0}$ is comprised of a certain number of planes that all intersect at a common line. The normal (row of A) of each plane is orthogonal to the line of intersection.



$\therefore$ by a theorem seen in class the solution set of $A\underline{x} = \underline{b}$ is

$\underline{x} =$ particular solution of $A\underline{x} = \underline{b}$ + general solution of $A\underline{x} = \underline{0}$

$= (3,-2,1) + t(1,3,-2)$ $t \in \mathbb{R}$

another point on $\mathscr{L}$ is when $t = 1$ $\underline{x} = (4,1,-1)$ and $\underline{d}_2 = (-2,-6,4) = -2(1,3,-2) = -2\underline{d}_1$

$\therefore \mathscr{L}$ and $\mathscr{L}'$ describe the same set.