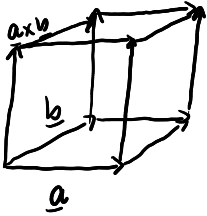


Books, watches, notes or cell phones are not allowed. The only calculators allowed are the Sharp EL-531\*\*\*. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** Given  $\mathbf{a}$  is a unit vector, the angle between  $\mathbf{a}$  and  $\mathbf{b}$  is  $\frac{\pi}{6}$ , and  $\|\mathbf{a} \times \mathbf{b}\| = 2$ .a. (3 marks) Find the volume of the parallelepiped defined by  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{a} \times \mathbf{b}$ .

$$\begin{aligned}
 \text{Volume} &= |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})| \text{ where } \mathbf{c} = \mathbf{a} \times \mathbf{b} \\
 &= |-\mathbf{c} \cdot (\mathbf{b} \times \mathbf{a})| \\
 &= |\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})| \\
 &= |(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{a} \times \mathbf{b})| \\
 &= \|\mathbf{a} \times \mathbf{b}\|^2 \\
 &= 2^2 \\
 &= 4
 \end{aligned}$$

b. (3 marks) Find the surface area of the parallelepiped defined by  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{a} \times \mathbf{b}$ .

$$\begin{aligned}
 \text{Surface Area} &= 2 \text{ parallelogram determined by } \mathbf{a} \text{ and } \mathbf{b} \\
 &\quad + 2 \text{ rectangle determined by } \mathbf{a} \text{ and } \mathbf{a} \times \mathbf{b} \\
 &\quad + 2 \text{ rectangle determined by } \mathbf{b} \text{ and } \mathbf{a} \times \mathbf{b} \\
 &= 2\|\mathbf{a} \times \mathbf{b}\| + 2\|\mathbf{a}\|\|\mathbf{a} \times \mathbf{b}\| + 2\|\mathbf{b}\|\|\mathbf{a} \times \mathbf{b}\| \\
 &= 2(2) + 2(1)(2) + 2(4)(2) \\
 &= 24
 \end{aligned}$$

$$\begin{aligned}
 \|\mathbf{a} \times \mathbf{b}\| &= \|\mathbf{a}\|\|\mathbf{b}\|\sin \frac{\pi}{6} \\
 2 &= 1\|\mathbf{b}\| \frac{1}{2} \\
 4 &= \|\mathbf{b}\|
 \end{aligned}$$

**Question 2.** (3 marks) Simplify:  $(\mathbf{u} + \mathbf{v}) \times (\mathbf{u} - \mathbf{v}) = \mathbf{u} \times (\mathbf{u} - \mathbf{v}) + \mathbf{v} \times (\mathbf{u} - \mathbf{v})$ 

$$\begin{aligned}
 &= \mathbf{u} \times \mathbf{u} - \mathbf{u} \times \mathbf{v} + \mathbf{v} \times \mathbf{u} - \mathbf{v} \times \mathbf{v} \\
 &= \mathbf{0} + \mathbf{v} \times \mathbf{u} + \mathbf{v} \times \mathbf{u} + \mathbf{0} \\
 &= 2\mathbf{v} \times \mathbf{u}
 \end{aligned}$$

**Bonus Question** (3 marks) Find the volume of the parallelepiped defined  $\mathbf{a} = (1, 2, 3, 4)$ ,  $\mathbf{b} = (1, 0, 1, 0)$  and  $\mathbf{c} = (0, 1, 0, 1)$ .