Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2024: **Quiz 14** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Determine whether $H = \{A \mid A \in \mathbb{M}_{n\times n}$ and the RREF of A is $I\}$, is a subspace of $\mathbb{M}_{n\times n}$

$\begin{bmatrix}1&0\\0&1\end{bmatrix} \in H$ since the RREF of I is I

$\begin{bmatrix}-1&0\\0&-1\end{bmatrix} \in H$ since $\begin{bmatrix}-1&0\\0&-1\end{bmatrix} \sim^{-R_1\to R_1}_{-R_2\to R_2} \begin{bmatrix}1&0\\0&1\end{bmatrix}$

$\begin{bmatrix}1&0\\0&1\end{bmatrix} + \begin{bmatrix}-1&0\\0&-1\end{bmatrix} = \begin{bmatrix}0&0\\0&0\end{bmatrix} \notin H$ since the RREF of 0 is not I.

Let $S = \{\underline{s}_1, \dots, \underline{s}_n\}$

Question 2. *(4 marks)* Prove: The span of a nonempty set S of vectors in V is the smallest subspace of V that contains S.

Suppose $\exists W$ subspace of V s.t. $W \subsetneq \text{span}(S)$. This implies $\exists\ \underline{v} = c_1\underline{s}_1 + \dots + c_n\underline{s}_n \in \text{span}(S)$ s.t. $\underline{v} \notin W$ ↯

Since $S \subset W$ and W is subspace, it must be closed under scalar multiplication i.e. $c_i\underline{s}_i \in W$
And it is also closed under addition i.e.
$c_1\underline{s}_1 + \dots + c_n\underline{s}_n \in W$

Question 3. *(4 marks)* If $\mathbf{v}_1, \dots, \mathbf{v}_n$ are linearly dependent nonzero vectors, then at least one vector $\mathbf{v}_k$ is a unique linear combination of $\mathbf{v}_1, \dots, \mathbf{v}_{k-1}$. Let $S_i = \{\underline{v}_1, \dots, \underline{v}_i\}$ where $1 \le i \le n$

$S_1 = \{\underline{v}_1\}$ is lin. ind since $\underline{v}_i \ne \underline{0}\ \forall i$
$S_2 = \{\underline{v}_1, \underline{v}_2\}$ is either lin. ind. or lin. dep. if lin. ind. we consider the next set S_3 (and so on). if S_2 is lin. dep then $\underline{v}_1$ is a multiple $\underline{v}_2$

⋮

Suppose $S_{i-1} = \{\underline{v}_1, \dots, \underline{v}_{i-1}\}$ is lin. ind. (then no vector in that set can be written as a lin. comb. of the others) and $\underline{S}_i$ is lin. dep.
then $\underline{v}_i$ can be written as a lin. comb. of S_{i-1}.
Suppose there exists two ways to express $\underline{v}_i$ as a lin. comb.

① $\underline{s}_i = c_1\underline{v}_1 + \dots + c_k\underline{v}_k + \dots + c_{i-1}\underline{v}_{i-1}$
② $\underline{s}_i = d_1\underline{v}_1 + \dots + d_k\underline{v}_k + \dots + d_{i-1}\underline{v}_{i-1}$
where $c_k \ne d_k$

①-② $\underline{0} = (c_1 - d_1)\underline{v}_1 + \dots + (c_k - d_k)\underline{v}_k + \dots + (c_{i-1} - d_{i-1})\underline{v}_{i-1}$
non trivial solution to above since $c_k - d_k \ne 0$
$\therefore S_{i-1}$ is lin. dep ↯ $\therefore$ Can be written in a unique way.