Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2024: **Quiz 15** name: ____________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Prove: If $S = \{(1,\, x_{1\,2}, \ldots,\, x_{1\,n-1}),\, (2,\, x_{2\,2}, \ldots,\, x_{2\,n-1}),\, (3,\, x_{3\,2}, \ldots,\, x_{3\,n-1}), \ldots,\, (n,\, x_{n\,2}, \ldots,\, x_{n\,n-1})\}$ where the vectors in S are in $\mathbb{R}^{n-1}$ and $x_{i\,j} \in \mathbb{R}$ then S is linearly dependent.

Question 2. *(5 marks)* Consider the subspace $W = \{p(x) \mid p(x) \in \mathbb{P}_3,\ \int_0^1 p(x)\,dx = 0 \text{ and } p'(1) = 0\}$ of $\mathbb{P}_3$.
Recall: $\int_0^1 a_0 + a_1x + a_2x^2 + a_3x^3\,dx = a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4}$ *and* $p'(1) = a_1 + 2a_2 + 3a_3$

a. *(4 marks)* Find a basis for W.

b. *(1 mark)* Determine the dimension of W.

c. *(2 marks)* Find a non-zero vector of W and find the coordinates of that vector relative to the basis found in part a.

d. *(1 mark)* Find a vector in $\mathbb{P}_3$ but not in W.