Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2024: **Quiz 15** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Prove: If $S = \{(1, x_{1\,2}, \ldots, x_{1\,n-1}),\ (2, x_{2\,2}, \ldots, x_{2\,n-1}),\ (3, x_{3\,2}, \ldots, x_{3\,n-1}), \ldots, (n, x_{n\,2}, \ldots, x_{n\,n-1})\}$ where the vectors in S are in $\mathbb{R}^{n-1}$ and $x_{i\,j} \in \mathbb{R}$ then S is linearly dependent.

The set is linearly dependent since the number of vectors in S is n which is greater than the $\dim(\mathbb{R}^{n-1}) = n-1$ of the enclosing space.

Question 2. *(5 marks)* Consider the subspace $W = \{p(x) \mid p(x) \in \mathbb{P}_3,\ \int_0^1 p(x)\,dx = 0 \text{ and } p'(1) = 0\}$ of $\mathbb{P}_3$.
Recall: $\int_0^1 a_0 + a_1x + a_2x^2\,dx = a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4}$ *and* $p'(1) = a_1 + 2a_2 + 3a_3$

a. *(4 marks)* Find a basis for W.

Lets find $p(x) \in W$, it must satisfy ① $a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4} = 0$ and ② $a_1 + 2a_2 + 3a_3 = 0$

$a_1 = -2a_2 - 3a_3$

sub ② into ① $\quad a_0 - \frac{2a_2 - 3a_3}{2} + \frac{a_2}{3} + \frac{a_3}{4} = 0$

$a_0 = \frac{2}{3}a_2 + \frac{1}{4}a_3 \qquad \therefore\ p(x) = \frac{2}{3}a_2 + \frac{5}{2}a_3 + (-2a_2 - 3a_3)x + a_2x^2 + a_3x^3$

$= a_2\underbrace{\left(\frac{2}{3} - 2x + x^2\right)}_{p_1(x)} + a_3\underbrace{\left(\frac{1}{4} - 3x + x^3\right)}_{p_2(x)}$

$\therefore\ \beta = \{p_1(x), p_2(x)\}$ spans W

and β is linearly independent since the vectors are not multiple of each other.

$\therefore\ \beta$ is a basis

b. *(1 mark)* Determine the dimension of W. $\dim(W) = 2$

c. *(2 marks)* Find a non-zero vector of W and find the coordinates of that vector relative to the basis found in part a.

$p_1(x) \in W \qquad (p_1(x))_\beta = (1, 0)$

d. *(1 mark)* Find a vector in $\mathbb{P}_2$ but not in W.

$p(x) = x \in \mathbb{P}_2$ but $p(x) \notin W$ since $p'(1) = 1 \neq 0$.