Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2024: **Quiz 1** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If the number of equations in a linear system is strictly more than the number of unknowns, then the system must be inconsistent.

False. $\begin{matrix} x=1 \\ 2x=2 \end{matrix}$ is a consistent system where $x=1$ is its solution set.

Question 2. *(3 marks)* In each of the following, find (if possible) conditions on k such that the system has one solution and infinitely many solutions. If any such k exists then find for each k the solution set of the system.

$$\begin{cases} x + ky = 0 \\ kx + y = 0 \end{cases}$$

If $k=1$ then both lines are identical. $\therefore$ infinitely many solutions.
And its solution set are all (x,y) that satisfy $x+y=0$.

let $y=t \quad t\in\mathbb{R}$

$x+t=0$

$x=-t$

$\therefore (x,y)=(-t,t) \quad t\in\mathbb{R}$.

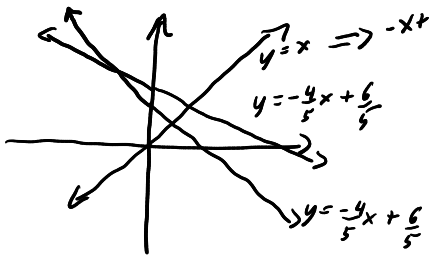
If $k\neq 1$ then both lines have different slopes but passes through the origin. $\therefore (x,y)=(0,0)$ is the solution set.

Question 3. *(2 marks)* Consider the following augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Find a row which can be added to the augmented matrix to make a new inconsistent system. Justify.

The above system consists of two lines $\left.\begin{matrix} x+2y=3 \\ 4x+5y=6 \end{matrix}\right\} \Rightarrow \begin{matrix} y=-\frac{1}{2}x+\frac{3}{2} \\ y=-\frac{4}{5}x+\frac{6}{5} \end{matrix}$



adding $[-1 \quad 1 \quad 0] \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 0 \end{bmatrix}$ creates a system of 3 lines that only have pairwise intersections

$\therefore$ no common intersection

$\therefore$ inconsistent.

Question 4. *(3 marks)* Illustrate **all** relative positions of lines in a consistent linear system consisting of three lines.