Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2024: **Quiz 2** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Find the value(s) of k, if any, for which the following system

$$\begin{cases} (2-k^2)x + (2-k^2)y + z = k \\ kx + y + kz = 4 \\ x + y + z = k \end{cases}$$

has

a. exactly one solutions,

b. infinitely many solutions,

c. no solutions.

$$\begin{bmatrix} 2-k^2 & 2-k^2 & 1 & k \\ k & 1 & k & 4 \\ 1 & 1 & 1 & k \end{bmatrix}$$

$$\sim_{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & 1 & 1 & k \\ k & 1 & k & 4 \\ 2-k^2 & 2-k^2 & 1 & k \end{bmatrix}$$

$$\sim \begin{matrix} -kR_1 + R_2 \to R_2 \\ -(2-k^2)R_1 + R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & 1 & 1 & k \\ 0 & 1-k & 0 & 4-k^2 \\ 0 & 0 & k^2-1 & k-k(2-k^2) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & k \\ 0 & 1-k & 0 & (2-k)(2+k) \\ 0 & 0 & (k-1)(k+1) & k(k^2-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & k \\ 0 & 1-k & 0 & (2-k)(2+k) \\ 0 & 0 & (k-1)(k+1) & k(k+1)(k-1) \end{bmatrix}$$

a) if $k \neq \pm 1$ then the system has a unique solution since # leading entries in var. col. = # var.

if $k=1$ then

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ no solution since leading 1 in constant column.

if $k=-1$ then

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ infinitely many solutions since # leading entries in variable column < # variable

Question 2. *(5 marks)* Find a sequence of elementary row operations that brings $\begin{bmatrix} b_1+c_1 & b_2+c_2 & b_3+c_3 \\ c_1+a_1 & c_2+a_2 & c_3+a_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{bmatrix}$ to $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$.

$$\sim_{-R_1+R_2 \to R_2} \begin{bmatrix} b_1+c_1 & b_2+c_2 & b_3+c_3 \\ a_1-b_1 & a_2-b_2 & a_3-b_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{bmatrix}$$

$$\sim_{R_3+R_2 \to R_2} \begin{bmatrix} b_1+c_1 & b_2+c_2 & b_3+c_3 \\ 2a_1 & 2a_2 & 2a_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{bmatrix}$$

$$\sim_{\frac{1}{2}R_2 \to R_2} \begin{bmatrix} b_1+c_1 & b_2+c_2 & b_3+c_3 \\ a_1 & a_2 & a_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{bmatrix}$$

$$\sim_{-R_2+R_3 \to R_3} \begin{bmatrix} b_1+c_1 & b_2+c_2 & b_3+c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

$$\sim_{-R_3+R_1 \to R_1} \begin{bmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

$$\sim_{R_1 \leftrightarrow R_2} \begin{bmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \sim_{R_2 \leftrightarrow R_3} \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$