Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2024: **Quiz 3** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If A and B are square matrices of the same order, then $\operatorname{tr}(AB)$ might be equal to $\operatorname{tr}(A)\operatorname{tr}(B)$.

Question 2.*(3 marks)* Find **all** matrices A that commute with $\underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{B}$.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$AB = BA$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\begin{bmatrix} 0 & a \\ 0 & c \end{bmatrix} = \begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{matrix} c = 0 \\ a = d \end{matrix}$

Let $\begin{matrix} b = t \\ d = s \end{matrix}$ $t, s \in \mathbb{R}$

$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} s & t \\ 0 & s \end{bmatrix}$ $s, t \in \mathbb{R}$

Question 3.*(3 marks)* Prove: If A is an $m \times n$ matrix and $A(BA)$ is defined, then B is an $n \times m$ matrix.

For the product BA to be defined, B needs the same # columns as rows of A.

$\therefore$ B has m columns.

The product $A \times (BA)$ to be defined, BA needs the same # rows as columns of A.

$\therefore$ BA has n rows

$\therefore$ B has n rows

$\therefore$ B is an $n \times m$ matrix.

Question 4.*(3 marks)* Use the following properties of the trace:

1. $\operatorname{tr}(A \pm B) = \operatorname{tr}(A) \pm \operatorname{tr}(B)$
2. $\operatorname{tr}(AB) = \operatorname{tr}(BA)$

to show that there does not exists matrices A and B such that $AB - BA = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. *Hint: Prove by contradiction.*

Suppose $\exists A, B$ s.t. $AB - BA = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\operatorname{tr}(AB - BA) = \operatorname{tr}\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right)$

$\operatorname{tr}(AB) - \operatorname{tr}(BA) = 5$ by 1

$0 = 5$ by 2 $\lightning$ $\therefore \nexists A, B$

[1] From or modified from a John Abbott final examination