Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2024: **Quiz 4** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Solve for the matrix A where

$$A^T \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} - 2(A^{-1})^T \right)^{-1}$$

Let $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$A^T B = (C - 2(A^{-1})^T)^{-1}$

$(A^T B)^{-1} = [(C - 2(A^{-1})^T)^{-1}]^{-1}$

$B^{-1}(A^T)^{-1} = C - 2(A^{-1})^T$

$B^{-1}(A^T)^{-1} + 2(A^{-1})^T = C$

$B^{-1}(A^T)^{-1} + 2(A^T)^{-1} = C$

$(B^{-1}+2I)(A^T)^{-1} = C$

$(B^{-1}+2I)(A^T)^{-1}(A^T) = C(A^T)$

$B^{-1}+2I = CA^T$

$C^{-1}(B^{-1}+2I) = C^{-1}CA^T$

$C^{-1}(B^{-1}+2I) = IA^T$

$C^{-1}(B^{-1}+2I) = A^T$

$(C^{-1}(B^{-1}+2I))^T = A$

$A = \left(\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\right)^T$

$A = \left(\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ -2 & 3 \end{bmatrix}\right)^T$

$A = \begin{bmatrix} 3 & 0 \\ -5 & 3 \end{bmatrix}^T$

$A = \begin{bmatrix} 3 & -5 \\ 0 & 3 \end{bmatrix}$

Question 2. *(4 marks)* Show that if A, B, and $A+B$ are invertible matrices with the same size, then $A(A^{-1}+B^{-1})B(A+B)^{-1} = I$. What does that equation imply about $A^{-1}+B^{-1}$? Justify.

$$\begin{aligned} A(A^{-1}+B^{-1})B(A+B)^{-1} &= (AA^{-1}+AB^{-1})B(A+B)^{-1} \\ &= (IB + AB^{-1}B)(A+B)^{-1} \\ &= (B + AI)(A+B)^{-1} \\ &= (A+B)(A+B)^{-1} \\ &= I \end{aligned}$$

From the equation we get

$A(A^{-1}+B^{-1})B(A+B)^{-1} = I$

$A^{-1}A(A^{-1}+B^{-1})B(A+B)^{-1} = A^{-1}I$

$I(A^{-1}+B^{-1})B(A+B)^{-1} = A^{-1}$

$(A^{-1}+B^{-1})B(A+B)^{-1}A = A^{-1}A = I$

we also get

$A(A^{-1}+B^{-1})B(A+B)^{-1} = I$

$A(A^{-1}+B^{-1})\underbrace{B(A+B)^{-1}(B(A+B)^{-1})^{-1}}_{I} = I(B(A+B)^{-1})^{-1}$

$A(A^{-1}+B^{-1}) = (B(A+B)^{-1})^{-1}$

$B(A+B)^{-1}A(A^{-1}+B^{-1}) = B(A+B)^{-1}(B(A+B)^{-1})^{-1}$

$B(A+B)^{-1}A(A^{-1}+B^{-1}) = I$

$\therefore$ $A^{-1}+B^{-1}$ is invertible and

$(A^{-1}+B^{-1})^{-1} = B(A+B)^{-1}A$

Question 3. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

A square matrix containing a row of zeros cannot be invertible.

True, Suppose the i^{th} row of A is all zeros and B be any square matrix of the same size of A.

Then the i^{th} row of AB is $[i^{th} \text{ row of } A]B = [0\ 0 \ldots 0]$

$\therefore$ impossible to obtain the identity

$\therefore$ A is singular