Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2024: **Quiz 5** name: Yann Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. Let A be a square matrix. If $A\mathbf{x} = A\mathbf{y}$ for distinct $\mathbf{x}$ and $\mathbf{y}$, then A cannot be invertible.

Question 2. *(5 marks)* Show that every matirix A can be factored as $A = UR$ where U is invertible and R is in reduced row-echelon form.

Perform Gauss-Jordan on A

$A \sim$ k elementary row operations $\sim R$ where R is the RREF of A.

For each of the k elementary row operations we can obtain an elementary matrix E_i s.t.

$$R = E_k \cdots E_2 E_1 A$$

since elementary matrices are invertible then so is their product

$$(E_k \cdots E_2 E_1)^{-1} R = (E_k \cdots E_2 E_1)^{-1} E_k \cdots E_2 E_1 A$$

$$(E_k \cdots E_2 E_1)^{-1} R = IA$$

$$U R = A$$

where $U = (E_k \cdots E_2 E_1)^{-1}$ an invertible matrix.

Question 3. *(5 marks)* Prove that if A and B are $m \times n$ matrices, then A and B are row equivalent if and only if A and B have the same reduced row echelon form.

[$\Rightarrow$] premise:

A and B are row equivalent

conclusion:

A and B have the same RREF

If we perform Gauss-Jordan on B:

$B \sim$ k elementary row operations $\sim R$

where R is the RREF of B.

And since A and B are row equivalent we have

$A \sim$ l elementary row operations $\sim B$

$\therefore A \sim$ l elem. row op $\sim$ k elem. row op $\sim R$

$\therefore$ A and B have the same RREF

[$\Leftarrow$] premise:

A and B have the same RREF

conclusion:

A and B are row equivalent

If we perform Gauss-Jordan on A and B:

$A \sim$ k elementary row operations $\sim R$

$B \sim$ l elementary row operations $\sim R$

the RREF is the same by the premise

Then

$A \sim$ k elem. row op $\sim R \sim$ the inverse elem. row op in reverse order $\sim B$

$\therefore$ A and B are row equivalent