Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2024: **Quiz 6** name: Y.Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.*(2 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If A and B are $n \times n$ matrices such that $A + B$ is symmetric, then A and B are symmetric.

False, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = A^T$ are not symmetric but $A + B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is symmetric

since $(A+B)^T = (A+B)$

Question 2.*(3 marks)* We showed in class that the product of symmetric matrices is symmetric if and only if the matrices commute. Is the product of commuting skew-symmetric matrices skew-symmetric? Justify.

Suppose A and B are skew-symmetric, that is $A^T = -A$ and $B^T = -B$. Lets also suppose that $AB = BA$. Then $(AB)^T = B^T A^T$

$= (-B)(-A)$

$= BA$

$= AB$

Then the product is not skew symmetric it is symmetric.

Question 3. *(5 marks)* Prove: If A and B are square matrices of the same size for which the system $A\mathbf{x} = \mathbf{b}$ is inconsistent for some column matrix $\mathbf{b}$ and $B\mathbf{x} = \mathbf{b}$ has a unique solution for all column matrix $\mathbf{b}$ then the reduced row echelon form of AB has at least one row of zeros.

By the equivalence theorem A is singular since $\exists \underline{b}$ s.t. $A\underline{x} = \underline{b}$ is inconsistent.
and B is invertible since $\forall \underline{b}$, $B\underline{x} = \underline{b}$ is consistent.

Suppose AB is invertible then $AB(AB)^{-1} = I$, let $C = B(AB)^{-1}$

$\therefore$ $\exists$ a matrix C s.t. $AC = I$ and C is a square matrix

$\therefore$ A is invertible by a theorem of §1.6

$\lightning$ since A is singular. $\therefore$ AB is singular

By a theorem seen in class AB (a square matrix) has a RREF that is I or has at least one row of zeros. But its not I since AB is singular (equivalence theorem).

$\therefore$ AB has at least one row of zeros.

Bonus. *(3 marks)* Prove: If A and B are lower triangular square matrices of the same size then AB is lower triangular.