

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531***. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. (1 mark each) Complete each of the following sentences with MUST, MIGHT, or CANNOT.

- a. If A is a 3×3 matrix and B is obtained from A by multiplying the first column by 4 and multiplying the third column by $\frac{3}{4}$, then $\det(B)$ _____ be equal to $3\det(A)$.

Question 1. (4 marks) Given the polynomial $p(x) = a + bx + cx^2 + dx^3 + x^4$, the matrix $C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a & -b & -c & -d \end{bmatrix}$ is called the companion matrix of $p(x)$. Show that $\det(xI - C) = p(x)$

Question 2. (3 marks) If A is an $n \times n$ matrix, the *characteristic polynomial* $c_A(x)$ of A is defined by $c_A(x) = \det(xI - A)$. The *Cayley-Hamilton Theorem* states that for any square matrix A , $c_A(x) = 0$ when evaluated at $x = A$. Prove the *Cayley-Hamilton Theorem* for 2×2 matrices. **Important Hint: First find the characteristic polynomial!**

Question 3. (5 marks) Only using elementary operations show that

$$\begin{vmatrix} a_1 + b_1t & a_2 + b_2t & a_3 + b_3t \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (1 - t^2) \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$