Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2024: **Quiz 8** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Show that $\det(I+2A^{-1}B^T) = 2^{n-1}\det(\frac{1}{2}A^T + B)$ where A and B are $n\times n$ matrices such that $\det(A) = 2$.

Since $\det(A) \neq 0$ then A is invertible by the equivalence theorem

$$
\begin{aligned}
LHS &= \det(A^{-1}A + 2A^{-1}B^T) \\
&= \det(2A^{-1}(\tfrac{1}{2}A + B^T)) \\
&= \det(2A^{-1})\det(\tfrac{1}{2}A + B^T) \\
&= 2^n \det A^{-1} \det((\tfrac{1}{2}A + B^T)^T) \\
&= 2^n \frac{1}{\det A} \det((\tfrac{1}{2}A)^T + (B^T)^T) \\
&= 2^n \frac{1}{2} \det(\tfrac{1}{2}A^T + B) \\
&= 2^{n-1}\det(\tfrac{1}{2}A^T + B) = RHS
\end{aligned}
$$

Question 2. *(3 marks)* **Without** using the formula $\det(\operatorname{adj}(A)) = (\det(A))^{n-1}$. Prove: If $\det(A) = 0$ then $\det(\operatorname{adj}(A)) = 0$. *Hint: Prove by contradiction.*

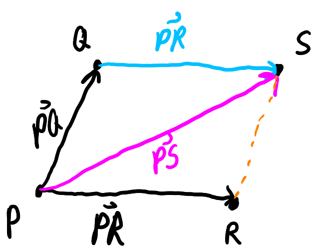
Suppose $\det(\operatorname{adj}(A)) \neq 0$. This implies that $\operatorname{adj}(A)$ is invertible by the equivalence theorem.

We know

$$
\begin{aligned}
A\operatorname{adj}(A) &= \det(A)I \\
A\operatorname{adj}(A) &= 0I \\
A\operatorname{adj}(A) &= 0 \\
A\operatorname{adj}(A)(\operatorname{adj}(A))^{-1} &= 0(\operatorname{adj}(A))^{-1} \\
AI &= 0 \\
A &= 0 \Rightarrow \operatorname{adj}(A) = 0 \quad \text{since } C_{ij} = 0 \ \forall i,j \\
&\Rightarrow \det(\operatorname{adj}(A)) = 0 \ ↯ \qquad \therefore \det(\operatorname{adj}(A)) = 0
\end{aligned}
$$

Question 3. *(3 marks)* Let $P(2,0,-1)$, $Q(-2,4,1)$, and $R(3,-1,0)$ be the vertices of a parallelogram with adjacent sides PQ and PR. Find the other vertex S.

Sketch:



$$
\begin{aligned}
\vec{PS} &= \vec{PQ} + \vec{PR} \\
\vec{OS} - \vec{OP} &= \vec{OQ} - \vec{OP} + \vec{OR} - \vec{OP} \\
\vec{OS} &= \vec{OQ} + \vec{OR} - \vec{OP} \\
\vec{OS} &= (-2,4,1) + (3,-1,0) - (2,0,-1) \\
\vec{OS} &= (-1,3,2)
\end{aligned}
$$

$\therefore S(-1,3,2)$