Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2024: **Quiz 9** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Let **u**, **v**, and **w** be pairwise orthogonal vectors.

a. *(3 marks)* Show that $||\mathbf{u}+\mathbf{v}+\mathbf{w}||^2 = ||\mathbf{u}||^2+||\mathbf{v}||^2+||\mathbf{w}||^2$.

$LHS = ||\underline{u}+\underline{v}+\underline{w}||^2$

$= (\underline{u}+\underline{v}+\underline{w})\cdot(\underline{u}+\underline{v}+\underline{w})$

$= \underline{u}\cdot\underline{u} + \underbrace{\underline{u}\cdot\underline{v}}_{0} + \underbrace{\underline{u}\cdot\underline{w}}_{0} + \underbrace{\underline{v}\cdot\underline{u}}_{0} + \underline{v}\cdot\underline{v} + \underbrace{\underline{v}\cdot\underline{w}}_{0} + \underbrace{\underline{w}\cdot\underline{u}}_{0} + \underbrace{\underline{w}\cdot\underline{v}}_{0} + \underline{w}\cdot\underline{w}$ since pairwise orthogonal.

$= ||\underline{u}||^2 + ||\underline{v}||^2 + ||\underline{w}||^2$

$= RHS$

b. *(3 marks)* If **u**, **v**, and **w** are all the same length, show that they all make the same angle with $\mathbf{u}+\mathbf{v}+\mathbf{w}$

Suppose $||\underline{u}|| = ||\underline{v}|| = ||\underline{w}|| = l$ which implies $||\underline{u}+\underline{v}+\underline{w}|| = \sqrt{||\underline{u}||^2+||\underline{v}||^2+||\underline{w}||^2} = \sqrt{l^2+l^2+l^2} = \sqrt{3}\,l$

① $\underline{u}\cdot(\underline{u}+\underline{v}+\underline{w}) = ||\underline{u}||\,||\underline{u}+\underline{v}+\underline{w}||\cos\theta_1$

$\underline{u}\cdot\underline{u} + \underbrace{\underline{u}\cdot\underline{v}}_{0} + \underbrace{\underline{u}\cdot\underline{w}}_{0} = l\sqrt{3}\,l\cos\theta_1$

$||\underline{u}||^2 + 0 + 0 = \sqrt{3}\,l^2\cos\theta_1$

$l^2 = \sqrt{3}\,l^2\cos\theta_1$

$\frac{1}{\sqrt{3}} = \cos\theta_1$

② Similarly

$\underline{v}\cdot(\underline{u}+\underline{v}+\underline{w}) = ||\underline{v}||\,||\underline{u}+\underline{v}+\underline{w}||\cos\theta_2$

$\vdots$

$\frac{1}{\sqrt{3}} = \cos\theta_2$

③ Similarly

$\underline{w}\cdot(\underline{u}+\underline{v}+\underline{w}) = ||\underline{w}||\,||\underline{u}+\underline{v}+\underline{w}||\cos\theta_3$

$\vdots$

$\frac{1}{\sqrt{3}} = \cos\theta_3$

Question 2. *(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If $\mathbf{u}\cdot\mathbf{v} = \mathbf{u}\cdot\mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.

False, Let $\underline{u} = (1,0)$, $\underline{v} = (0,1)$, $\underline{w} = (0,2)$ we have $\underline{u}\cdot\underline{v} = 0$, $\underline{u}\cdot\underline{w} = 0$ but $\underline{v} \neq \underline{w}$

b. If **a** and **b** are nonzero orthogonal vectors, then for every nonzero vector **u**, we have $\text{proj}_{\mathbf{a}}(\text{proj}_{\mathbf{b}}(\mathbf{u})) = \mathbf{0}$.

True,

$LHS = \frac{\underline{a}\cdot \text{proj}_{\underline{b}}\underline{u}}{\underline{a}\cdot\underline{a}}\underline{a}$

$= \frac{\underline{a}\cdot\frac{\underline{b}\cdot\underline{u}}{\underline{b}\cdot\underline{b}}\underline{b}}{\underline{a}\cdot\underline{a}}\underline{a}$

$= \frac{\frac{\underline{b}\cdot\underline{u}}{\underline{b}\cdot\underline{b}}\,\underline{a}\cdot\underline{b}}{\underline{a}\cdot\underline{a}}\underline{a} = \frac{\frac{\underline{b}\cdot\underline{u}}{\underline{b}\cdot\underline{b}}\,0}{\underline{a}\cdot\underline{a}}\underline{a} = 0\underline{a} = \underline{0}$