Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 10** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Given the planes $\mathscr{P}_1 : x+3y-z=5$ and $\mathscr{P}_2 : 2x-5y+z=7$. Determine the equation of the line of intersection and find the angle between the given planes.

$$\begin{bmatrix}1&3&-1&5\\2&-5&1&7\end{bmatrix} \underset{-2R_1+R_2\to R_2}{\sim} \begin{bmatrix}1&3&-1&5\\0&-11&3&-3\end{bmatrix} \underset{11R_1\to R_1}{\sim} \begin{bmatrix}11&33&-11&55\\0&-11&3&-3\end{bmatrix} \underset{3R_2+R_1\to R_1}{\sim} \begin{bmatrix}11&0&-2&46\\0&-11&3&-3\end{bmatrix}$$

$$\underset{\substack{\frac{1}{11}R_1\to R_1\\ -\frac{1}{11}R_2\to R_2}}{\sim} \begin{bmatrix}1&0&-2/11&46/11\\0&1&-3/11&3/11\end{bmatrix}$$

Let $z=t$ $t\in\mathbb{R}$

$x = \frac{46}{11} + \frac{2}{11}t$

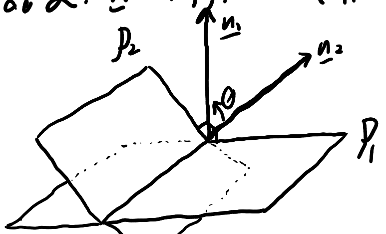
$y = \frac{3}{11} + \frac{3}{11}t$

$\therefore (x,y,z) = \left(\frac{3}{11}+\frac{3}{11}t, \frac{46}{11}+\frac{2}{11}t, t\right)$

$= \left(\frac{3}{11}, \frac{46}{11}, 0\right) + t\left(\frac{3}{11}, \frac{2}{11}, 1\right)$

$t\in\mathbb{R}$

$\therefore \mathscr{L}: \underline{x} = (x,y,z) = \left(\frac{46}{11}+\frac{2}{11}t, \frac{-3}{11}-\frac{3}{11}t, t\right) = \left(\frac{46}{11}, \frac{-3}{11}, 0\right) + t\left(\frac{2}{11}, \frac{-3}{11}, 1\right)$



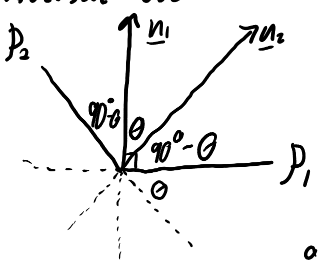
$\underline{n}_1\cdot\underline{n}_2 = \|\underline{n}_1\|\|\underline{n}_2\|\cos\theta$

$(1,3,-1)\cdot(2,-5,1) = \sqrt{1^2+3^2+(-1)^2}\sqrt{2^2+(-5)^2+1^2}\cos\theta$

$\cos\theta = \dfrac{-14}{\sqrt{11}\sqrt{30}}$

$\theta \approx 140^\circ$

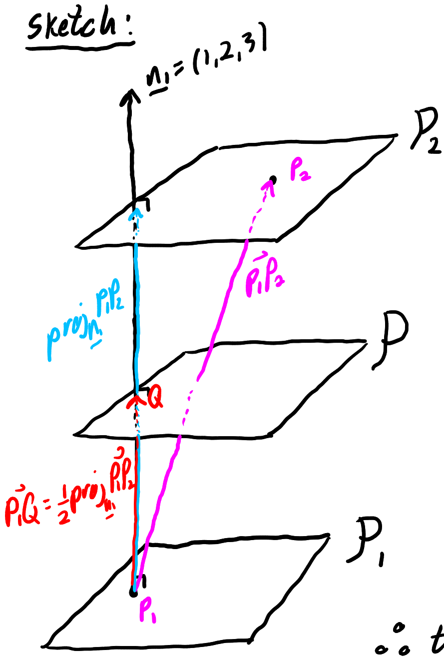
Transversal cut:



$\therefore \theta$ is 140° or 40°

Question 2. *(5 marks)* Find the equation of the plane $\mathscr{P}$ which is equidistant from $\mathscr{P}_1 : x+2y+3z=0$ and $\mathscr{P}_2 : x+2y+3z=6$.

Sketch:



Lets find a point Q on P.

P_1 contains the origin O. $\therefore P_1(0,0,0)$

P_2 contains $P_2(1,1,1)$.

So $\vec{P_1P_2} = \vec{OP_2} - \vec{OP_1} = (1,1,1)-(0,0,0) = (1,1,1)$

$\vec{P_1Q} = \frac{1}{2}proj_{\underline{n}_1}\vec{P_1P_2}$

$\vec{OQ}-\vec{OP_1} = \frac{1}{2}\dfrac{\underline{n}_1\cdot\vec{P_1P_2}}{\underline{n}_1\cdot\underline{n}_1}\underline{n}_1$

$\vec{OQ} = \frac{1}{2}\dfrac{(1,2,3)\cdot(1,1,1)}{(1,2,3)\cdot(1,2,3)}(1,2,3)$

$\vec{OQ} = \frac{1}{2}\,\frac{6}{14}(1,2,3)$

$\therefore Q = \frac{3}{14}(1,2,3)$

$\therefore$ the point-normal equation of the plane is

$\left(x-\frac{3}{14}\right) + 2\left(y-\frac{6}{14}\right) + 3\left(z-\frac{9}{14}\right) = 0.$

Or

Q = Midpoint of P_1 and P_2

$= \frac{1}{2}(P_1+P_2)$

$= \frac{1}{2}((0,0,0)+(1,1,1))$

$= \left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$

and $x+2y+3z=d$

$\frac{1}{2}+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)=d$

$\frac{6}{2}=d$

$3=d$

$\therefore x+2y+3z=3$