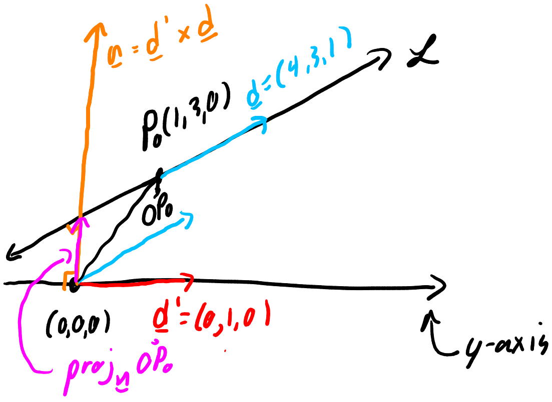
Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 11** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Find the distance between the line $\mathscr{L} : (x, y, z) = (1, 3, 0) + t(4, 3, 1)$ and the y-axis.



The equation of the line on the y-axis is $\underline{x} = (0,0,0) + t(0,1,0)$.

$\vec{OP_0} = (1,3,0)$

$\underline{n} = \underline{d}' \times \underline{d} = \left(\begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix}, - \begin{vmatrix} 0 & 4 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 4 \\ 1 & 3 \end{vmatrix} \right) = (1,0,-4)$

$\begin{matrix} 0 & 4 \\ 1 & 3 \\ 0 & 1 \end{matrix}$

$$\text{proj}_{\underline{n}} \vec{OP_0} = \frac{\underline{n} \cdot \vec{OP_0}}{\underline{n} \cdot \underline{n}} \underline{n} = \frac{(1,0,-4)\cdot(1,3,0)}{(1,0,-4)\cdot(1,0,-4)} (1,0,-4) = \frac{1}{17}(1,0,-4)$$

$$\text{distance} = \|\text{proj}_{\underline{n}} \vec{OP_0}\| = \left\|\frac{1}{17}(1,0,-4)\right\| = \left|\frac{1}{17}\right| \|(1,0,-4)\| = \frac{1}{17}\sqrt{1+0+16} = \frac{\sqrt{17}}{17}$$

Question 2. Given two planes:

$$\begin{aligned} \mathscr{P}_1 &: \quad x + z = 1 \\ \mathscr{P}_2 &: \quad y + z = 1 \end{aligned}$$

a. *(1 mark)* Give a point of intersection of the two planes, by inspection.

The point $(0,0,1)$ satisfy both equations.

b. *(1 mark)* Give a geometrical argument to explain why the intersection of the two planes is a line.

$P_1 \nparallel P_2$ since $\underline{n}_1 \nparallel \underline{n}_2$ because $\nexists k$ s.t. $\underline{n}_2 = k\underline{n}_1$

c. *(2 marks)* Find the direction vector for the intersection of the two planes without solving for the solution set. Justify.

As seen in a theorem in class the lines must be orthogonal to both normals.

$\therefore \underline{d} = \underline{n}_1 \times \underline{n}_2 = \left(\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}, - \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right) = (-1,-1,1)$

$\begin{matrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{matrix}$

d. *(1 mark)* Find the solution set of the system of linear equations determined by $\mathscr{P}_1$ and $\mathscr{P}_2$ by only using part a) and part c).

$\underline{x} = (0,0,1) + t(-1,-1,1)$ where $t \in \mathbb{R}$.

Bonus Question. *(3 marks)* Given a *Yann plane segment* defined as $\mathbf{x} = (1,0,2) + s(1,1,1) + t(2,1,3)$ where $(s,t) \in [-1,2] \times [-2,0]$. Find the area of the Yann plane segment.