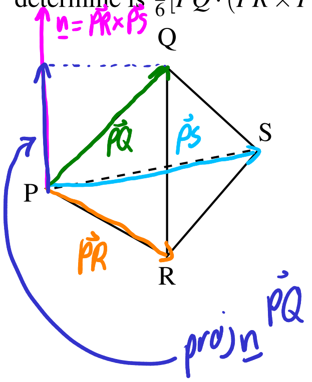
Dawson College: **Linear Algebra** (Science): 201-NYC-05-S8: Winter 2024: **Quiz 12** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Let P, Q, R, and S be four points, not all on one plane, as in the diagram. Show that the volume of the pyramid they determine is $\frac{1}{6}[\vec{PQ}\cdot(\vec{PR}\times\vec{PS})]$. Hint: The volume of a pyramid with base area A and height h is $\frac{1}{3}Ah$.



$$
\begin{aligned}
\text{Volume} &= \frac{1}{3}Ah\\
&= \frac{1}{3}\frac{1}{2}\|\underline{n}\|\|\text{proj}_{\underline{n}}\vec{PQ}\|\\
&= \frac{1}{6}\|\underline{n}\|\left\|\frac{\vec{PQ}\cdot\underline{n}}{\underline{n}\cdot\underline{n}}\underline{n}\right\|\\
&= \frac{1}{6}\|\underline{n}\|\left|\frac{\vec{PQ}\cdot\underline{n}}{\underline{n}\cdot\underline{n}}\right|\|\underline{n}\|\\
&= \frac{1}{6}\cancel{\|\underline{n}\|}\frac{|\vec{PQ}\cdot\underline{n}|\cancel{\|\underline{n}\|}}{\cancel{\|\underline{n}\|^2}}\\
&= \frac{1}{6}|\vec{PQ}\cdot\underline{n}|\\
&= \frac{1}{6}|\vec{PQ}\cdot(\vec{PR}\times\vec{PS})|
\end{aligned}
$$

Question 2.

a. *(3 marks)* Show that $\mathbf{w}\cdot(\mathbf{u}\times\mathbf{v}) = \mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \mathbf{v}\cdot(\mathbf{w}\times\mathbf{u})$ holds for all vectors $\mathbf{w}$, $\mathbf{u}$, and $\mathbf{v}$.

$$
\begin{aligned}
\underline{w}\cdot(\underline{u}\times\underline{v}) &= \begin{vmatrix} w_1 & w_2 & w_3\\ u_1 & u_2 & u_3\\ v_1 & v_2 & v_3\end{vmatrix}\\
&= -\begin{vmatrix} v_1 & v_2 & v_3\\ u_1 & u_2 & u_3\\ w_1 & w_2 & w_3\end{vmatrix} \quad R_1 \leftrightarrow R_3\\
&= \begin{vmatrix} u_1 & u_2 & u_3\\ v_1 & v_2 & v_3\\ w_1 & w_2 & w_3\end{vmatrix} \quad R_1 \leftrightarrow R_2\\
&= \underline{u}\cdot(\underline{v}\times\underline{w})
\end{aligned}
$$

$$
\begin{aligned}
&= -\begin{vmatrix} v_1 & v_2 & v_3\\ u_1 & u_2 & u_3\\ w_1 & w_2 & w_3\end{vmatrix} \quad R_1 \leftrightarrow R_2\\
&= \begin{vmatrix} v_1 & v_2 & v_3\\ w_1 & w_2 & w_3\\ u_1 & u_2 & u_3\end{vmatrix} \quad R_2 \leftrightarrow R_3\\
&= \underline{v}\cdot(\underline{w}\times\underline{u})
\end{aligned}
$$

b. *(3 marks)* Show that $\mathbf{v}-\mathbf{w}$ and $\overbrace{(\mathbf{u}\times\mathbf{v})+(\mathbf{v}\times\mathbf{w})+(\mathbf{w}\times\mathbf{u})}^{\underline{z}}$ are orthogonal.

$$(\underline{v}-\underline{w})\cdot[(\underline{u}\times\underline{v})+(\underline{v}\times\underline{w})+(\underline{w}\times\underline{u})]$$

$$= \underbrace{\underline{v}\cdot(\underline{u}\times\underline{v})}_{0} + \underbrace{\underline{v}\cdot(\underline{v}\times\underline{w})}_{0} + \underline{v}\cdot(\underline{w}\times\underline{u}) - \underline{w}\cdot(\underline{u}\times\underline{v}) - \underbrace{\underline{w}\cdot(\underline{v}\times\underline{w})}_{0} - \underbrace{\underline{w}\cdot(\underline{w}\times\underline{u})}_{0}$$

$$= \underline{v}\cdot(\underline{w}\times\underline{u}) - \underline{v}\cdot(\underline{w}\times\underline{u}) \quad \text{by part a)}$$

$$= 0$$

$\therefore$ $\underline{v}-\underline{w}$ and $\underline{z}$ are orthogonal.