Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 14** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Determine whether $H = \{(a,\ b) \mid a, b \in \mathbb{R} \text{ and } ab = 0\}$, is a subspace of $\mathbb{R}^2$

$(1,0) \in H$ since $1(0) = 0$

$(0,1) \in H$ since $0(1) = 0$

But $(1,0) + (0,1) = (1,1) \notin H$ since $1(1) = 1 \neq 0$

Question 2. *(4 marks)* Let V be the subspace of vectors parallel to the line $\mathbf{x} = (1,\ 2,\ 3) + t(1,\ 1,\ 1)$ where $t \in \mathbb{R}$, and let W be the subspace spanned by $(1,\ 1,\ 0)$ and $(0,\ 1,\ 1)$. Find a vector $\mathbf{v}$ in V and a vector $\mathbf{w}$ in W for which $\mathbf{v} + \mathbf{w} = (1,\ 0,\ 1)$

$V = \text{span}(\{(1,1,1)\})$

$W = \text{span}(\{(1,1,0),(0,1,1)\})$

$(1,0,1) = c_1(1,1,1) + c_2(1,1,0) + c_3(0,1,1)$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \underset{\substack{-R_1+R_2 \to R_2 \\ -R_1+R_3 \to R_3}}{\sim} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \underset{R_2 \leftrightarrow R_3}{\sim} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\underset{-R_3+R_2 \to R_2}{\sim} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \underset{\substack{R_2+R_1 \to R_1 \\ -R_2 \to R_2}}{\sim} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$\therefore (1,0,1) = 2(1,1,1) + (-1)(1,1,0) + (-1)(0,1,1)$

$= \underbrace{(2,2,2)}_{\in V} + \underbrace{(-1,-2,-1)}_{\in W}$

Question 3. *(4 marks)* Prove that in $\mathbb{P}_2$ every set with more than three vectors is linearly dependent.

Let $p_1(x) = a_{01} + a_{11}x + a_{21}x^2$

$p_2(x) = a_{02} + a_{12}x + a_{22}x^2$

$\vdots$

$p_n(x) = a_{0n} + a_{1n}x + a_{2n}x^2$ and $n > 3$

$0 + 0x + 0x^2 = c_1 p_1(x) + c_2 p_2(x) + \cdots + c_n p_n(x)$

$$\begin{bmatrix} a_{01} & a_{02} & \cdots & a_{0n} & 0 \\ a_{11} & a_{12} & \cdots & a_{1n} & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 \end{bmatrix}$$

Since $n > 3$ the system will have at least $n-3$ parameters $\therefore$ not only the trivial solution. $\therefore$ linearly dependent.