Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 15** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Prove: If $p(x) \in \mathbb{P}_{n-1}$ then $\{p(x),\ p'(x),\ p''(x), \ldots,\ p^{(n)}(x)\}$ is linearly dependent.

The set is linearly dependant since the set contains $n+1$ vectors which is greater than the $\dim(\mathbb{P}_{n-1}) = n-1+1 = n$ of the enclosing space.

Question 2. *(5 marks)* Consider the subspace $W = \{X \mid X \in \mathscr{M}_{3\times3} \text{ and } X^T = X \text{ and } \operatorname{trace}(X) = 0\}$ of $\mathscr{M}_{3\times3}$.

a. *(4 marks)* Find a basis for W.

For $X \in W$, $X = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & g \end{bmatrix}$ and $a+d+g=0$
$g = -a-d$

$\therefore X = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & -a-d \end{bmatrix} \in W$

$= a \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{M_1} + b \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{M_2} + c \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{M_3} + d \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{M_4} + e \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{M_5}$

$\therefore \beta = \{M_1, M_2, M_3, M_4, M_5\}$ spans W

And β is linearly independent since $0 = c_1 M_1 + c_2 M_2 + c_3 M_3 + c_4 M_4 + c_5 M_5$
$\Rightarrow c_1 = 0$
$c_2 = 0$
$c_3 = 0$
$c_4 = 0$
$c_5 = 0$

$\therefore \beta$ is a basis of W

b. *(1 mark)* Determine the dimension of W. $\dim(W) = 5$

c. *(2 marks)* Find a non-zero vector of W and find the coordinates of that vector relative to the basis found in part a.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \in W$ and $\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \right)_\beta = (1,0,0,0,0)$

d. *(1 mark)* Find a vector in $\mathscr{M}_{3\times3}$ but not in W.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in M_{3\times3}$ but $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \notin W$ since $\operatorname{trace}\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) = 1 \neq 0$