Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 1** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If the number of equations in a linear system is strictly more than the number of unknowns, then the system must be consistent.

False, $\begin{cases} x+y=1 \\ x+y=2 \\ x+y=3 \end{cases}$ The system is inconsistent since $\nexists x,y$ which adds to both 1 and 2.

Question 2. *(3 marks)* In each of the following, find (if possible) conditions on k such that the system has no solution and one solution.

$$\begin{cases} x + ky = 2 \\ kx + y = 4 \end{cases}$$

If $K=0$ then the system becomes $\begin{cases} x=2 \\ y=4 \end{cases}$ $\therefore$ unique solution $(x,y)=(2,4)$

If $K \neq 0$ then the system can be rewritten as

$y = -\frac{1}{K}x + \frac{2}{K}$

$y = -Kx + 4$

and if the slopes of both lines are different then we obtain a unique solution

$-\frac{1}{K} \neq -K$

$1 \neq K^2$

$\pm 1 \neq K$ $\therefore$ unique solution if $K \neq \pm 1$

If $K = \pm 1$ then the two lines have the same slope but different y-intercept. $\therefore$ no solution

Question 3. *(2 marks)* Consider the following augmented matrix of a consistent linear system.

$$\left[\begin{matrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{matrix}\right] \Rightarrow \begin{matrix} x+2y=3 \\ 2x+4y=6 \end{matrix}$$

Find a row which can be added to the augmented matrix to make a new system with infinitely many solutions. Justify.

The two lines of the system are identical. Adding an other identical line will make a new system which still has infinitely many solutions. i.e. $3x+6y=9$

$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ has infinitely many solutions.

Question 4. *(3 marks)* Illustrate **all** relative positions of lines in a linear system with a unique solution consisting of four lines.