Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 2** name: ~~Y. Lamontagne~~

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Find the value(s) of k, if any, for which the following system

$$\begin{cases} x - 3y - z = 2 \\ -2x + (k^2+2)y - kz = -2-3k \\ 3x - 9y + (k^2+k-9)z = 2k+2 \end{cases}$$

$$\begin{bmatrix} 1 & -3 & -1 & 2 \\ -2 & k^2+2 & -k & -2-3k \\ 3 & -9 & k^2+k-9 & 2k+2 \end{bmatrix}$$

has

a. exactly one solutions,
b. infinitely many solutions,
c. no solutions.

$$\sim \begin{matrix} 2R_1+R_2\to R_2 \\ -3R_1+R_3\to R_3 \end{matrix} \begin{bmatrix} 1 & -3 & -1 & 2 \\ 0 & k^2-4 & -2-k & 2-3k \\ 0 & 0 & k^2+k-6 & 2k-4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & -1 & 2 \\ 0 & (k+2)(k-2) & -(k+2) & 2-3k \\ 0 & 0 & (k+3)(k-2) & 2(k-2) \end{bmatrix}$$

Lets look at all values of k which make an entry vanish in the matrix.

$k=2$: the augmented matrix becomes

$$\begin{bmatrix} 1 & -3 & -1 & 2 \\ 0 & 0 & -4 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The system is consistent since no leading entry in constant column. And has infinitely many solutions since # leading entries < # variable

$k=-2$: $\begin{bmatrix} 1 & -3 & -1 & 2 \\ 0 & 0 & 0 & 8 \\ 0 & 0 & -4 & -8 \end{bmatrix}$ has no solution since it has a leading entry in constant column.

$k=-3$: $\begin{bmatrix} 1 & -3 & -1 & 2 \\ 0 & 5 & 1 & 11 \\ 0 & 0 & 0 & -10 \end{bmatrix}$ "

$k \neq \pm 2, -3$ has a unique solution since # leading entry in var. col = # var.

Question 2. *(5 marks)* Show that the reduced row echelon form of $\begin{bmatrix} p & 0 & a \\ b & 0 & 0 \\ q & c & r \end{bmatrix}$ where $abc \neq 0$ is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

$abc \neq 0$ implies $a \neq 0$, $b \neq 0$, $c \neq 0$

$$\sim_{R_1 \leftrightarrow R_2} \begin{bmatrix} b & 0 & 0 \\ p & 0 & a \\ q & c & r \end{bmatrix} \sim_{\frac{1}{b}R_1 \to R_1} \begin{bmatrix} 1 & 0 & 0 \\ p & 0 & a \\ q & c & r \end{bmatrix} \sim_{\substack{-pR_1+R_2\to R_2 \\ -qR_1+R_3\to R_3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & a \\ 0 & c & r \end{bmatrix}$$

$$\sim_{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & r \\ 0 & 0 & a \end{bmatrix} \sim_{\frac{1}{a}R_3 \to R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & r \\ 0 & 0 & 1 \end{bmatrix} \sim_{-rR_3+R_2\to R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sim_{\frac{1}{c}R_2 \to R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$