Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 3** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. Let A be a square matrix such that not all entries are zero. All entries of AA <u>might</u> be equal to zero.

b. If B has a column of zeros and the product AB is defined then AB <u>must</u> have a column of zeros.

c. If A has a row of zeros and the product AB is defined then AB <u>must</u> have a row of zeros.

Question 2.*(4 marks)* Find **all** matrices A where

$$A + 3A^T = \begin{bmatrix} 12 & 2 \\ -10 & 4 \end{bmatrix}$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A + 3A^T = \begin{bmatrix} 12 & 2 \\ -10 & 4 \end{bmatrix}$

$\begin{bmatrix} 12 & 2 \\ -10 & 4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + 3\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$\begin{bmatrix} 12 & 2 \\ -10 & 4 \end{bmatrix} = \begin{bmatrix} 4a & b+3c \\ c+3b & 4d \end{bmatrix}$

$12 = 4a \Rightarrow a = 3$

$4 = 4d \Rightarrow d = 1$

① $2 = b + 3c$

② $-10 = 3b + c$

$-3$① $+$ ②:

$-16 = -8c$

$2 = c$ sub into ① $2 = b + 3(2) \Rightarrow b = -4$ $\therefore A = \begin{bmatrix} 3 & -4 \\ 2 & 1 \end{bmatrix}$

Question 3.*(3 marks)* Prove: If AB and BA are both defined, then AB and BA are square matrices.

Suppose A is a $m \times n$ matrix and B is a $p \times q$ matrix

Since AB is defined then $n = p$

Since BA is defined then $m = q$

$\therefore B$ is a $n \times m$ matrix which implies that $A_{m\times n}B_{n\times m}$ is an $m \times m$ matrix

$B_{n\times m}A_{m\times n}$ is an $n \times n$ matrix

[1] From or modified from a John Abbott final examination