Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 3** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(5 marks)* Solve for the matrix X if $(D+XB)^{-1}XA = (ED)^{-1}$. Assume that all matrices are $n \times n$ and invertible as needed.

$$(D+XB)(D+XB)^{-1}XA = (D+XB)(ED)^{-1}$$
$$I\ XA = (D+XB)D^{-1}E^{-1}$$
$$XA = DD^{-1}E^{-1} + XBD^{-1}E^{-1}$$
$$XA = IE^{-1} + XBD^{-1}E^{-1}$$
$$XA - XBD^{-1}E^{-1} = E^{-1}$$
$$X(A-BD^{-1}E^{-1}) = E^{-1}$$
$$X(A-BD^{-1}E^{-1})(A-BD^{-1}E^{-1})^{-1} = E^{-1}(A-BD^{-1}E^{-1})^{-1}$$
$$XI = \quad ''$$
$$X = \quad ''$$

Question 2.*(4 marks)* Show that if A is a square matrix such that $A^k = 0$ for some positive integer k, then the matrix $I-A$ is invertible and $(I-A)^{-1} = I + A + A^2 + \cdots + A^{k-1}$

premise:
- A is square
- $A^k = 0$ for some $k \in \mathbb{Z}$

conclusion:
- $I - A$ is invertible
- $(I-A)^{-1} = I + A + A^2 + \cdots + A^{k-1}$

Lets show that $\exists B$ s.t.

① $(I-A)B = I$

② $B(I-A) = I$

Let $B = I + A + A^2 + \cdots + A^{k-1}$

$(I-A)(I+A+A^2+\cdots+A^{k-1})$

$= II + IA + IA^2 + \cdots + IA^{k-1} - AI - AA - AA^2 - \cdots - A^k$

$= I + \cancel{A} + \cancel{A^2} + \cdots + \cancel{A^{k-1}} - \cancel{A} - \cancel{A^2} - \cancel{A^3} - \cdots - A^k$

$= I - A^k$

$= I$ by premise since $A^k = 0$

$(I+A+A^2+\cdots+A^{k-1})(I-A)$

$= I + AI + A^2I + \cdots + A^{k-1}I - IA - AA - A^2A - \cdots - A^k$

$= I + \cancel{A} + \cancel{A^2} + \cdots + \cancel{A^{k-1}} - \cancel{A} - \cancel{A^2} - \cancel{A^3} - \cdots - A^k$

$= I - A^k$

$= I$ by premise since $A^k = 0$

$\therefore$ $I - A$ is invertible and

$(I-A)^{-1} = I + A + A^2 + \cdots + A^{k-1}$

Question 3.*(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

The sum of two invertible matrices of the same size must be invertible.

False, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is invertible since $ad - bc = 1(1) - 0(0) = 1 \neq 0$

$B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ is " " $ad - bc = (-1)(-1) - 0(0) = 1 \neq 0$

But $A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is not invertible $ad - bc = (0)(0) - (0)(0) = 0$

[1]based on a WeBWorK problem