Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 5** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Let A and B, be 3×4 matrices such that B is obtained from the matrix A using the following elementary row operations:

a. Switch the first and 3rd row.

b. Multiply the 2nd row by $\frac{1}{3}$

c. Add twice the first row to the 3rd row.

Find a matrix C, such that $A = CB$.

E_1:

$I_3 \sim_{R_1 \leftrightarrow R_3} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = E_1$

E_2:

$I_3 \sim_{\frac{1}{3}R_2 \to R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_2$

E_3:

$I_3 \sim_{2R_1 + R_3 \to R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = E_3$

$B = E_3E_2E_1A$

$(E_3E_2E_1)^{-1}B = \underbrace{(E_3E_2E_1)^{-1}E_3E_2E_1}_{I}\,A$

$E_1^{-1}E_2^{-1}E_3^{-1}B = A$

$A = CB$

where $C = E_1^{-1}E_2^{-1}E_3^{-1}$

$= E_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$= E_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Question 3.*(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. Every elementary matrix is invertible.

True,

The RREF of any elementary matrix is the identity. The identity can be obtained by applying the inverse elementary row operation from which the elementary matrix was obtained. It follows by the Equivalence theorem that A is invertible since its RREF is I.

Or even better not to lead to lead to circular proofs in our notes:

An elementary matrix obtained using the inverse elementary row operation of the elementary row operation from which an other elementary matrix was obtained. These two matrices when multiplied will give the identity. Hence elementary matrices are invertible.

b. An expression of an invertible matrix as a product of elementary matrices is unique.

False,

$I = I$

and

$I = I \cdot I$

I is an elementary matrix since $I \sim_{1 \cdot R_1 \to R_1} I$