

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531***. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. (2 marks each) Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

- a. If A and B are $n \times n$ matrices such that $A + B$ is symmetric, then A and B are symmetric.

Question 2. (3 marks) We showed in class that the product of symmetric matrices is symmetric if and only if the matrices commute. Is the product of commuting skew-symmetric matrices skew-symmetric? Justify.

Question 3. (5 marks) Prove: If A and B are square matrices of the same size for which the system $A\mathbf{x} = \mathbf{b}$ is inconsistent for some column matrix \mathbf{b} and $B\mathbf{x} = \mathbf{b}$ has a unique solution for all column matrix \mathbf{b} then the reduced row echelon form of AB has at least one row of zeros.

Bonus. (3 marks) Prove: If A and B are lower triangular square matrices of the same size then AB is lower triangular.