Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 6** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Prove: If $A^TA = A$, then A is symmetric and $A = A^2$.

premise:

$A^TA = A$

conclusion:

① $A^T = A$

② $A^2 = A$

① LHS $= A^T$

$= (A^TA)^T$ from premise

$= A^T(A^T)^T$

$= A^TA$

$=$ RHS

② LHS $= A^2$

$= AA$

$= A^TA$ since A is symmetric by ①

$= A$

$=$ RHS

Question 2. *(2 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If A is a symmetric and skew-symmetric matrix then $A = 0$.

True, premise:

- A is symmetric $A^T = A$
- A is skew-symmetric $A^T = -A$

conclusion:

$A = 0$

By premise $A = -A$

$2A = 0$

$A = 0$

Question 3. *(5 marks)* Prove: If A is a square matrix for which the system $A\mathbf{x} = \mathbf{b}$ has infintely many solutions for some column matrix $\mathbf{b}$ and A is row equivalent to B then $B\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

premise:

- $A\underline{x} = \underline{b}$ has ∞ many sol. for some b
- A is row equivalent to B

conclusion:

$B\underline{x} = \underline{0}$ has ∞ many solutions

By the equivalence theorem the RREF of A is not I since . By a theorem seen in class then R the RREF of A must at least have a row of zeros because it is a square matrix.

There exists a sequence of elementary row operations such that

$B \sim k$ elementary row operations $\sim A$

since .

$\therefore$ $[B|0] \sim k$ elem. row op. $\sim [A|0] \sim$ Gauss-Jordan $\sim [R|0]$

$\therefore$ $B\underline{x} = \underline{0}$ has ∞ many solutions since R is a square matrix with at least a row of zeros which implies that #leading 1 $<$ # var.

Bonus. *(3 marks)* Prove: If A is an $m \times n$ matrix and B is an $n \times r$ matrix then $(AB)^T = B^TA^T$.