Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 7** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

If A is a 4×4 matrix and B is obtained from A by interchanging the first two rows and then interchanging the last two rows, then $\det(A)$ __must__ be equal to $\det(B)$.

Question 1. *(4 marks)* Find all the values of x for which

$$\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & x & x \\ 1 & x & 0 & x \\ 1 & x & x & 0 \end{vmatrix} = -6$$

$$-6 = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} + a_{14}C_{14}$$

$$-6 = -\begin{vmatrix} 1 & x & x \\ 1 & 0 & x \\ 1 & x & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & x \\ 1 & x & x \\ 1 & x & 0 \end{vmatrix} - \begin{vmatrix} 1 & 0 & x \\ 1 & x & 0 \\ 1 & x & x \end{vmatrix}$$

$$-6 = -[a_{21}C_{21} + \overbrace{a_{22}C_{22}}^{0} + a_{23}C_{23}] + [a_{11}C_{11} + \overbrace{a_{12}C_{12}}^{0} + a_{13}C_{13}] - [a_{11}C_{11} + \overbrace{a_{12}C_{12}}^{0} + a_{13}C_{13}]$$

$$-6 = -\left[-\begin{vmatrix} x & x \\ x & 0 \end{vmatrix} - \overbrace{\begin{vmatrix} 1 & x \\ 1 & x \end{vmatrix}}^{0}\right] + \left[1\begin{vmatrix} x & x \\ x & 0 \end{vmatrix} + x\overbrace{\begin{vmatrix} 1 & x \\ 1 & x \end{vmatrix}}^{0}\right] - \left[1\begin{vmatrix} x & 0 \\ x & x \end{vmatrix} + x\overbrace{\begin{vmatrix} 1 & x \\ 1 & x \end{vmatrix}}^{0}\right]$$

$$-6 = -x^2 - x^2 - x^2$$

$$-6 = -3x^2$$

$$2 = x^2$$

$$x = \pm\sqrt{2}$$

Question 2. *(2 marks)* Find b if

$$\begin{vmatrix} 1 & 2 & 3 & x \\ 1 & 3 & 4 & y \\ 0 & 2 & 3 & z \\ 0 & 0 & 3 & 4 \end{vmatrix} = ax + by + cz + d$$

If we do a cofactor expansion along the 4^{th} column there will only one cofactor contributing to the coefficient of y.

$$\therefore\ y\,C_{24} = y(-1)^{2+4}\begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{vmatrix} = 3!\,y$$

$$\therefore\ b = 3! = 6$$

Question 3. *(5 marks)* Only use elementary operations to show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$$

$$\text{LHS} \underset{\substack{-aR_1 + R_2 \to R_2 \\ -a^2R_1 + R_3 \to R_3}}{=} \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & (b-a)(b+a) & (c-a)(c+a) \end{vmatrix}$$

$$\underset{-(b+a)R_2 + R_3 \to R_3}{=} \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & (c-a)(c+a) - (c-a)(b+a) \end{vmatrix}$$

$$= 1(b-a)[(c-a)(c+a) + (c-a)(b+a)]$$

$$= (b-a)(c-a)[c+a-(b+a)]$$

$$= (b-a)(c-a)(c-b)$$