Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 8** name: Y.Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Let A and B be two 3×3 matrices such that $\det(A) = 4$ and $\det(B) = -2$. Find $\det(4B^{-1}A^{-1} + \operatorname{adj}(AB))$.

since $\det(A) \neq 0 \neq \det(B)$, A and B are invertible by the equivalence theorem.

$\therefore$ AB is invertible because it is the product of invertible matrices

$\therefore$ $(AB)^{-1} = \frac{1}{\det(AB)}\operatorname{adj}(AB)$

$\operatorname{adj}(AB) = \det(AB)(AB)^{-1} = \det(A)\det(B)B^{-1}A^{-1} = -8B^{-1}A^{-1}$

$\therefore$ $\det(4B^{-1}A^{-1} + (-8)B^{-1}A^{-1})$

$= \det(-4B^{-1}A^{-1})$

$= (-4)^3\det(B^{-1})\det(A^{-1})$

$= (-4)^3\frac{1}{\det B}\frac{1}{\det A} = (-4)^{\cancel{3}\,2}\frac{1}{\cancel{-2}}\frac{1}{\cancel{4}} = 8$

Question 2. *(3 marks)* Let A be a $n\times n$ matrix, such that n is odd if $A^2A^T + A^TA^2 = 0$, show that the system $A\mathbf{x} = \mathbf{0}$ has non-trivial solutions.

$A^2A^T + A^TA^2 = 0$

$A^2A^T = -A^TA^2$

$\det(A^2A^T) = \det(-A^TA^2)$

$\det(A^2)\det(A^T) = (-1)^n\det(A^TA^2)$

$(\det A)^2\det A = -1\det(A^T)\det(A^2)$ since n is odd

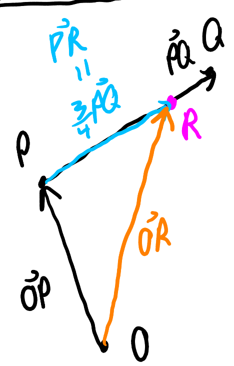
$(\det A)^3 = -\det A(\det A)^2$

$(\det A)^3 = -(\det A)^3$

$\therefore$ $\det A = 0$ $\therefore$ by the equivalence theorem $A\underline{x} = \underline{0}$ does not only have the trivial solution.

Question 3. *(3 marks)* Given $P(2,3,-2)$ and $Q(7,-4,1)$. Find the point on the line segment connecting the points P and Q that is $\frac{3}{4}$ of the way from P to Q.

Sketch:



$\vec{OR} = \vec{OP} + \frac{3}{4}\vec{PQ}$

$\vec{OR} = \vec{OP} + \frac{3}{4}[\vec{OQ} - \vec{OP}]$

$\vec{OR} = \frac{1}{4}\vec{OP} + \frac{3}{4}\vec{OQ}$

$\vec{OR} = \frac{1}{4}(2,3,-2) + \frac{3}{4}(7,-4,1)$

$\vec{OR} = \left(\frac{23}{4}, \frac{-9}{4}, \frac{1}{4}\right)$

$\therefore$ $R\left(\frac{23}{4}, \frac{-9}{4}, \frac{1}{4}\right)$