Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 9** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Let $\mathbf{u}$ be a unit vector, and let $\mathbf{v}$ be a vector such that $||\mathbf{v}|| = \sqrt{6}$, and $\mathbf{u} \cdot \mathbf{v} = -\frac{1}{2}$. Find $||2\mathbf{u} - 3\mathbf{v}||$.

$$\begin{aligned}
\|2\underline{u} - 3\underline{v}\|^2 &= (2\underline{u} - 3\underline{v}) \cdot (2\underline{u} - 3\underline{v}) \\
&= (2\underline{u})(2\underline{u}) - (2\underline{u}) \cdot (3\underline{v}) - (3\underline{v}) \cdot (2\underline{u}) + (3\underline{v}) \cdot (3\underline{v}) \\
&= 4\underline{u} \cdot \underline{u} - 6\underline{u} \cdot \underline{v} - 6\underline{v} \cdot \underline{u} + 9\underline{v} \cdot \underline{v} \\
&= 4\|\underline{u}\|^2 - 6\underline{u} \cdot \underline{v} - 6\underline{u} \cdot \underline{v} + 9\|\underline{v}\|^2 \\
&= 4(1)^2 - 12\underline{u} \cdot \underline{v} + 9(\sqrt{6})^2 \\
&= 58 - 12\left(-\tfrac{1}{2}\right) \\
&= 64
\end{aligned}$$

$\therefore \|2\underline{u} - 3\underline{v}\| = \sqrt{64} = 8$

Question 2.*(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If $\mathbf{u} \cdot \mathbf{v} = 0$, then either $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.

False, let $\underline{u} = (1,0)$ and $\underline{v} = (0,1)$, we have that $\underline{u} \cdot \underline{v} = 0$
but $\underline{u} \neq \underline{0}$ and $\underline{v} \neq \underline{0}$

b. If $\mathbf{a}$ and $\mathbf{u}$ are nonzero vectors, then $\text{proj}_{\mathbf{a}}(\text{proj}_{\mathbf{a}}(\mathbf{u})) = \text{proj}_{\mathbf{a}}(\mathbf{u})$.

True, LHS $= \dfrac{\underline{a} \cdot \text{proj}_{\underline{a}}(\underline{u})}{\underline{a} \cdot \underline{a}}\,\underline{a}$

$$\begin{aligned}
&= \frac{\underline{a} \cdot \frac{\underline{a} \cdot \underline{u}}{\underline{a} \cdot \underline{a}}\,\underline{a}}{\underline{a} \cdot \underline{a}}\,\underline{a} \\
&= \frac{\underline{u} \cdot \underline{a}}{\underline{a} \cdot \underline{a}} \frac{\underline{a} \cdot \underline{a}}{\underline{a} \cdot \underline{a}}\,\underline{a} \\
&= \frac{\underline{a} \cdot \underline{u}}{\underline{a} \cdot \underline{a}}\,\underline{a} \\
&= \text{proj}_{\underline{a}}\,\underline{u} \\
&= \text{RHS}
\end{aligned}$$