Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2024: **Quiz 6** name: _____

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Prove: If $A^TA = A$, then A is symmetric and $A = A^2$.

Question 2.*(2 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If A is a symmetric and skew-symmetric matrix then $A = 0$.

Question 3. *(5 marks)* Prove: If A is a square matrix for which the system $A\mathbf{x} = \mathbf{b}$ has infintely many solutions for some column matrix $\mathbf{b}$ and A is row equivalent to B then $B\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

Bonus. *(3 marks)* Prove: If A is an $m \times n$ matrix and B is an $n \times r$ matrix then $(AB)^T = B^TA^T$.