Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2025: **Quiz 1** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. Consider a system of linear equations with augmented matrix A. If there are no solutions then A has no row of zeros.

False, $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ has a row of zeros but $0x+0y=1$ is inconsistent.

b. If each equation in a consistent linear system is multiplied through by a constant c, then all solutions to the new system can be obtained by multiplying solutions from the original system by c.

False, the system $\begin{matrix} x+y=1 \\ y=1 \end{matrix}$ has solution $(x,y)=(0,1)$

but $\begin{matrix} cx+cy=c \\ cy=c \end{matrix}$ still has solution $(x,y)=(0,1)$ provided $c\neq 0$

Question 2. *(3 marks)* Find (if possible) conditions on a and b such that the system has no solution, one solution, and infinitely many solutions. Justify.

$$\begin{cases} x \quad -2y = 1 \\ ax \quad +by = 5 \end{cases}$$

For no solution there needs to be no points in common between the two lines. So their slopes must be equal but not their x or y-intercept. There is also the case $a=0=b$ where the last equation is inconsistent. Note that if $a=0$ or $b=0$ then we have a unique solution.

So suppose $a\neq 0, b\neq 0$ $\quad y=\frac{1}{2}x-\frac{1}{2}$

$y=-\frac{a}{b}x+\frac{5}{b}$ we need $\frac{1}{2}=\frac{-a}{b}$ and $\frac{-1}{2}\neq+\frac{5}{b}$

$b=-2a$ $\quad b\neq -10$

$\therefore$ no sol. if $(a,b)=(0,0)$ or $b=-2a$ and $b\neq-10$.

For unique solution the slopes of the two lines must be different i.e. $b\neq-2a$

For ∞-many solutions the lines must be identical.

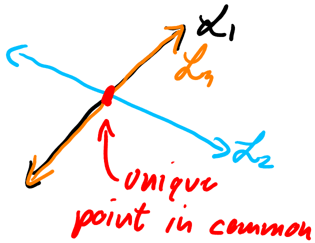
i.e. $a=5$, $b=-10$.

Question 3. *(2 marks)* Consider the following augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 2 & 4 & 6 \end{bmatrix} \quad \begin{array}{l} \mathcal{L}_1: \; x+2y=3 \\ \mathcal{L}_2: \; 2x+3y=4 \\ \mathcal{L}_3: \; 2x+4y=6 \end{array}$$

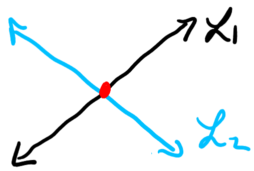
Find a row which can be removed from the augmented matrix to make a new system with two equations which has the same number of solutions. Justify.

Note that $\mathcal{L}_1$ and $\mathcal{L}_3$ are identical

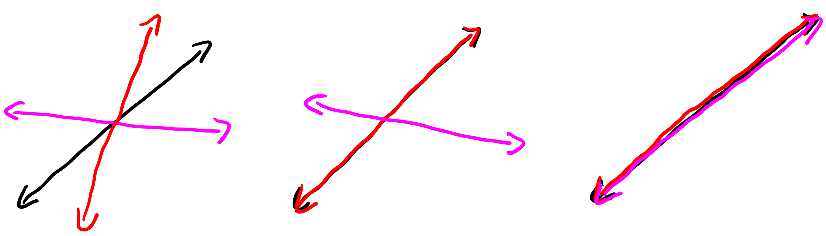


and that $\mathcal{L}_1$ and $\mathcal{L}_2$ have different slopes, same for $\mathcal{L}_1$ and $\mathcal{L}_3$. $\therefore$ there is a unique solution.

If we remove $\mathcal{L}_3$ we still have a unique solution $\therefore$ we can remove row 3.



Question 4. *(2 marks)* Illustrate **all** relative positions of lines in a consistent linear system consisting of three lines.



Question 5. *(3 marks)* Find the solution set of the following equation $4x_1 - x_2 + x_3 + \lambda x_4 = 1$. Find λ if a particular solution is $(x_1, x_2, x_3, x_4) = (1, 1, 1, 1)$.

Let $x_2 = s$
$x_3 = t \qquad s, t, r \in \mathbb{R}$
$x_4 = r$

$4x_1 - s + t + \lambda r = 1$
$4x_1 = 1 + s - t - \lambda r$
$x_1 = \frac{1}{4} + \frac{1}{4}s - \frac{1}{4}t - \frac{\lambda}{4}r$

$\therefore (x_1, x_2, x_3, x_4) = \left(\frac{1}{4} + \frac{1}{4}s - \frac{1}{4}t - \frac{\lambda}{4}r,\ s, t, r\right)$
$s, t, r \in \mathbb{R}$

If its a solution is satisfies the system.

$\therefore 4(1) - 1 + 1 + \lambda(1) = 1$
$\lambda = -3$