Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2025: **Quiz 2** name: _____

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If A is an $m \times n$ matrix then $\text{tr}(A^T A) \geq 0$.

Question 3. *(5 marks)* Find the reduced row echelon form of $\begin{bmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{bmatrix}$ where $c \neq a$ and $b \neq a$.

Question 2. *(3 marks)* Find all 2×2 matrices M such that $MA - AM = 0$ where $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

Question 3.[1] *(6 marks)* Find the values, if any, of h and k for which the following system has:

$$\begin{cases} x & + & 3y & + & 2z & = & k+5 \\ -x & + & (h-1)y & + & (h^2-6)z & = & k-1 \\ 3x & + & 9y & + & (h^2-h)z & = & k^2+3k+11 \end{cases}$$

Exactly one solution, no solutions, infinitely many solutions.

Bonus Question. *(3 marks)* If A, B and C are matrices such that the operations are defined, show that $A(BC) = (AB)C$.

[1] From a John Abbott Final Examination