Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2025: **Quiz 3** name: _____

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If A is row equivalent to a product of elementary matrices, then the system $A\mathbf{x} = \mathbf{b}$ has a unique solution for all $\mathbf{b}$.

Question 2. *(5 marks)* Solve for the matrix X in the following equation:

$$\left(\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} X^{-1} + I\right)^T = \left(\begin{bmatrix} -1 & 4 \\ 1 & -3 \end{bmatrix} X^T\right)^{-1}$$

Question 3. *(3 marks)* Let A be an $n \times n$ matrix such that $A^n = nA - (n-1)I$ where $n \neq 1$. Show that A is invertible, and find A^{-1} in terms of A and I_n.

Question 4. *(5 marks)* Express

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

and A^{-1} as a product of elementary matrices.