Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2025: **Quiz 4** name: ____________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If A is any skew-symmetric matrix then A^2 is skew-symmetric matrix.

Question 2. *(5 marks)* Let A and B be two 3×3 matrices such that $\det(A) = \begin{vmatrix} a+2b & c & b \\ d+2e & f & e \\ g+2h & i & h \end{vmatrix} = -2$ where $a, d, g \neq 0$, and $\det(B) = 3$. Find the following: $adg \begin{vmatrix} 2 & 2 & 2 \\ \frac{b}{a} & \frac{e}{d} & \frac{h}{g} \\ \frac{c}{a} & \frac{f}{d} & \frac{i}{g} \end{vmatrix}$.

Question 3. *(3 marks)* Prove that if $A^T A = A$, then A is symmetric and $A = A^2$.

Question 4. *(5 marks)* If A is an $n \times n$ matrix, the *characteristic polynomial* $c_A(x)$ of A is defined by $c_A(x) = \det(xI - A)$.

a. *(5 marks)* Find the eigenvalues λ of $A = \begin{bmatrix} 2 & 2 & -2 \\ 1 & 3 & -1 \\ -1 & 1 & 1 \end{bmatrix}$. That is, find the values of λ for which $c_A(\lambda) = 0$.

b. *(3 marks bonus)* Find the eigenvector for a non-zero eigenvalue found in part a. That is, find a non-trivial solution $\mathbf{x}$ for each λ where $A\mathbf{x} = \lambda\mathbf{x}$.