Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2025: **Quiz 5** name: _____

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If A and B are $n \times n$ matrices and $BA^2 + B^2A$ is invertible then $A + B$ is invertible.

Question 2. *(3 marks)* Let A denote an invertible $n \times n$ matrix where $n \geq 2$. Show that $\text{adj}(\text{adj}(\text{A})) = (\det A)^{n-2}A$.

Question 3. *(5 marks)* Let A, B, and C denote the three vertices of a triangle. If E is the midpoint of side BC, show that: $\vec{AE} = \frac{1}{2}(\vec{AB} + \vec{AC})$.

Question 4.[1] Let $\vec{u}$ and $\vec{v}$ be vectors in $\mathbb{R}^n$. Given: $||\vec{u}|| = 5$, $||\vec{u} + 2\vec{v}|| = \sqrt{2}$, $\vec{v}$ and $\vec{u} + 3\vec{v}$ are both unit vectors, and the angle between $\vec{u} + 2\vec{v}$ and $\vec{u} + 3\vec{v}$ is $\pi/4$.

a. *(3 marks)* Find $\vec{u} \cdot \vec{v}$.

b. *(2 marks)* Find $||\vec{u} + \vec{v}||$.

[1] From or modified from a John Abbott final examination