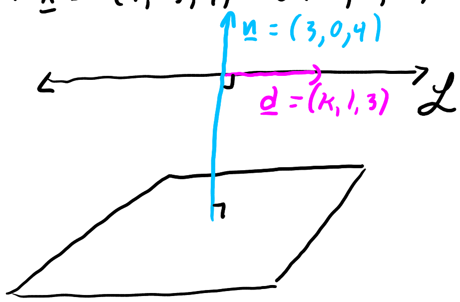
Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2025: **Quiz 6** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Consider the lines $\mathscr{L}: \begin{cases} x = kt+7 \\ y = t-3 \\ z = 3t+4 \end{cases}$, $t \in \mathbb{R}$ and the plane $\mathscr{P}: \ 3x+4z=7$

a. *(3 marks)* Determine the values of k, if any, for which $\mathscr{L}$ is parallel to $\mathscr{P}$.

$\mathscr{L}: \underline{x} = (7,-3,4) + t(k,1,3)$



$\mathscr{L}$ is parallel to $\mathscr{P}$ if $\underline{n} \perp \underline{d}$, so if $\underline{n}\cdot\underline{d} = 0$

$0 = (3,0,4)\cdot(k,1,3)$

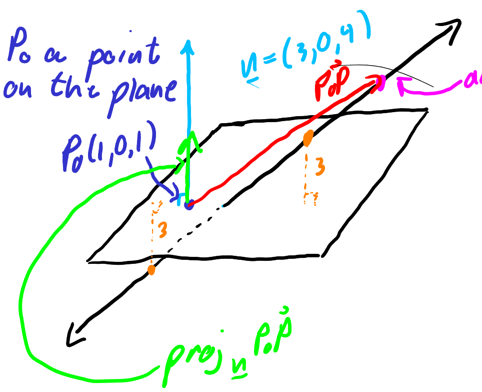
$0 = 3k+12$

$k = -4$

$\therefore$ parallel if $k=-4$

b. *(5 marks)* If $k=1$ find the points on the line $\mathscr{L}$ that are 3 units away from the plane $\mathscr{P}$.

Since $k \neq -4$, the $\mathscr{L}$ and $\mathscr{P}$ are not parallel



arbitrary point on $\mathscr{L}$: $P(t+7, t-3, 3t+4)$

We are looking for the values of t for which $\|proj_{\underline{n}} \vec{P_0P}\| = 3$

$\vec{P_0P} = \vec{OP} - \vec{OP_0}$

$= (t+7, t-3, 3t+4) - (1,0,1)$

$= (t+6, t-3, 3t+3)$

$$3 = \left\| \frac{(3,0,4)\cdot(t+6,t-3,3t+3)}{(3,0,4)\cdot(3,0,4)}(3,0,4) \right\|$$

$$3 = \left\| \frac{3t+18+12t+12}{9+16}(3,0,4) \right\|$$

$$3 = \left\| \frac{15t+30}{25}(3,0,4) \right\|$$

$$3 = \frac{|3t+6|}{5}\|(3,0,4)\|$$

$$3 = \frac{|3t+6|}{5}\sqrt{9+16}$$

$$3 = \frac{|3t+6|}{\cancel{5}}\cancel{5}$$

$$\pm 3 = 3t+6$$

$$\frac{-6\pm 3}{3} = t$$

$$t = -3 \text{ or } -1$$

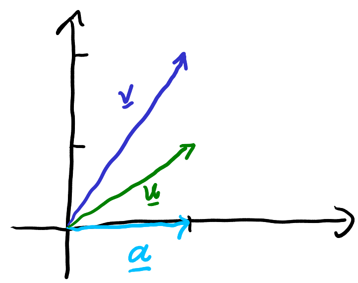
$\therefore P(-1+7, -1-3, 3(-1)+4) = P(6,-4,1)$

and $P(-3+4, -3-3, 3(-3)+4) = P(1,-6,-5)$

Question 2. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

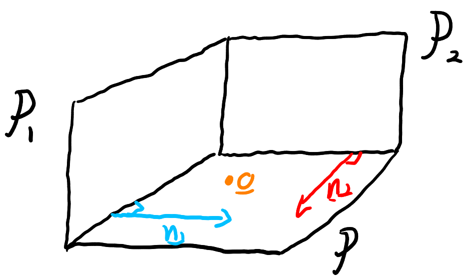
If the relationship $\text{proj}_{\mathbf{a}}(\mathbf{u}) = \text{proj}_{\mathbf{a}}(\mathbf{v})$ hold for some nonzero vector $\mathbf{a}$, then $\mathbf{u} = \mathbf{v}$.

False, let $\underline{a} = (1,0)$
$\underline{u} = (1,1)$
$\underline{v} = (1,2)$



We have that
$\text{proj}_{\underline{a}}(\underline{u}) = \text{proj}_{\underline{a}}(\underline{v}) = (1,0)$
but $\underline{u} \neq \underline{v}$

Question 3. *(2 marks)* Find the parametric equation of the plane which is orthogonal to both $\mathscr{P}_1 : x+y+z = 1$ and $\mathscr{P}_2 : x+2y+z = 3$ and passes through the origin.



$P: \underline{x} = \underline{P_0} + s\underline{d_1} + t\underline{d_2} \quad s,t \in \mathbb{R}$
$= \underline{0} + s\underline{n_1} + t\underline{n_2}$
$= s(1,1,1) + t(1,2,1)$

Question 4. *(3 marks)* Consider the system with equations: $x+y+z = b_1$, $x+2y+cz = b_2$ and $x+3y+dz = b_3$ where b_1, b_2, b_3, c, d are fixed real values, $P(1,1,1)$ satisfies all three equations and the solution set of the corresponding homogeneous linear system is $\mathbf{x} = t(2,-1,-1)$ where $t \in \mathbb{R}$.

Using a clearly labelled sketch give a geometrical interpretation of the linear system and its solution set, and the corresponding homogeneous linear system and its solution set.

We notice that the three equations are planes in $\mathbb{R}^3$ with normals
$\underline{n_1} = (1,1,1)$ The three planes have different inclinations since the normals are
$\underline{n_2} = (1,2,c)$ not multiples of each other
$\underline{n_3} = (1,3,d)$