

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531\*\*\*. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** (5 marks) Consider the points  $A(2, -2, 4)$ ,  $B(4, -1, 1)$ ,  $C(3, -1, 2)$ . and  $D(1, -1, 1 + \lambda)$ . Find all  $\lambda$  such that the volume of the parallelepiped determined by  $\vec{AB}$ ,  $\vec{AC}$ , and  $\vec{AD}$  is 2022.

**Question 2.** (5 marks) Find the points on the following skew lines  $\mathcal{L}_1 : \begin{cases} x = 4 + 2t \\ y = 2 + 3t \\ z = 2 + t \end{cases}$ , and  $\mathcal{L}_2 : \begin{cases} x = -3 + 2s \\ y = 1 - s \\ z = 1 + s \end{cases}$ ,  $s, t \in \mathbb{R}$  which are closest to each other.

**Question 3.** Consider the set

$$V = \{(x, y) \mid x \geq 0 \text{ and } y \geq 0\}$$

under the following operations:

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 y_2) \quad k(x, y) = (kx, y)$$

- a. (2 marks) Does  $V$  contain a zero vector? If so find it. Justify.
- b. (2 marks) Does  $V$  contain the additive inverse (negative of the vector in the sense of a vector space) of  $\vec{v} = (3, 2)$ ? If so find it. Justify.
- c. (1 mark) Is  $V$  a vector space? Justify.

**Question 4.** (5 marks) Let  $W = \{f \mid f(-x) = -f(x)\}$ . Determine whether  $W$  is a subspace of  $V = \{f \mid f : \mathbb{R} \rightarrow \mathbb{R}\}$ .

**Bonus.** (3 marks) Sketch  $r(t) = (\sin t, \cos t, t)$  where  $t \in \mathbb{R}$ .