Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2025: **Quiz 7** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Consider the points $A(2,-2,4)$, $B(4,-1,1)$, $C(3,-1,2)$. and $D(1,\ -1,\ 1+\lambda)$. Find all λ such that the volume of the parallelepiped determined by $\vec{AB}$, $\vec{AC}$, and $\vec{AD}$ is 2022.

$$\vec{AB} = \vec{OB} - \vec{OA} = (4,-1,1) - (2,-2,4) = (2,1,-3)$$
$$\vec{AC} = \vec{OC} - \vec{OA} = (3,-1,2) - (2,-2,4) = (1,1,-2)$$
$$\vec{AD} = \vec{OD} - \vec{OA} = (1,-1,1+\lambda) - (2,-2,4) = (-1,1,-3+\lambda)$$

$$\text{Volume} = |\vec{AB}\cdot(\vec{AC}\times\vec{AD})|$$

$$2022 = \left|\begin{vmatrix} 2 & 1 & -3 \\ 1 & 1 & -2 \\ -1 & 1 & -3+\lambda \end{vmatrix}\right|$$

$$2022 = \begin{matrix} \\ -R_1+R_2\to R_2 \\ -R_1+R_3\to R_3 \end{matrix}\left|\begin{vmatrix} 2 & 1 & -3 \\ -1 & 0 & 1 \\ -3 & 0 & \lambda \end{vmatrix}\right|$$

$$2022 = |a_{21}C_{21} + a_{22}C_{22} + a_{32}C_{32}|$$

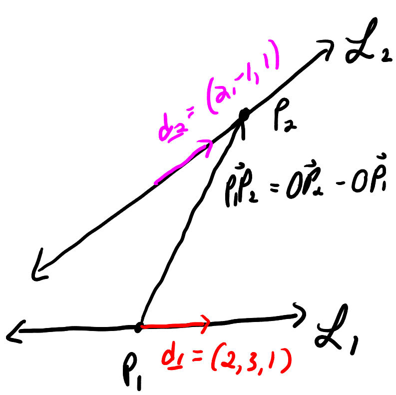
$$2022 = \left|(-1)(1)\begin{vmatrix} -1 & 1 \\ -3 & \lambda \end{vmatrix}\right|$$

$$\pm 2022 = -1(-\lambda+3)$$

$$\pm 2022 = \lambda - 3$$

$$\lambda = 3 \pm 2022$$
$$= 2025 \text{ or } -2019$$

Question 2. *(5 marks)* Find the points on the following skew lines $\mathscr{L}_1 : \begin{cases} x = 4+2t \\ y = 2+3t \\ z = 2+t \end{cases}$, and $\mathscr{L}_2 : \begin{cases} x = -3+2s \\ y = 1-s \\ z = 1+s \end{cases}$, $s,\ t \in \mathbb{R}$ which are closest to each other.



$$\vec{P_1P_2} = \vec{OP_2} - \vec{OP_1} = (-3+2s,1-s,1+s) - (4+2t,2+3t,2+t)$$
$$= (-7+2s-2t,-1-s-3t,-1+s-t)$$

Point on $\mathscr{L}_1$ when $t=-1$

$x = 4+2(-1) = 2$
$y = 2+3(-1) = -1$ $\therefore P_1(2,-1,1)$
$z = 2+(-1) = 1$

Point on $\mathscr{L}_2$ when $s=2$

$x = -3+2(2) = 1$
$y = 1-2 = -1$ $\therefore P_2(1,-1,3)$
$z = 1+2 = 3$

$$0 = \underline{d_1}\cdot\vec{P_1P_2} = (2,3,1)\cdot(-7+2s-2t,-1-s-3t,-1+s-t) = 2(-7+2s-2t)+3(-1-s-3t)+(-1+s-t)$$
$$0 = -14+4s-4t-3-3s-9t-1+s-t$$
$$0 = -18+2s-14t$$
$$9 = s-7t$$

$$0 = \underline{d_2}\cdot\vec{P_1P_2} = (2,-1,1)\cdot \quad ''$$
$$0 = 2(-7+2s-2t)-(-1-s-3t)+1(-1+s-t)$$
$$0 = -14+4s-4t+1+s+3t-1+s-t$$
$$0 = -14+6s-2t$$
$$7 = 3s-t$$

$$\begin{bmatrix} 1 & -7 & 9 \\ 3 & -1 & 7 \end{bmatrix} \sim_{-3R_1+R_2\to R_2} \begin{bmatrix} 1 & -7 & 9 \\ 0 & 20 & -20 \end{bmatrix} \sim_{\frac{1}{20}R_2\to R_2} \begin{bmatrix} 1 & -7 & 9 \\ 0 & 1 & -1 \end{bmatrix} \sim_{7R_2+R_1\to R_1} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

$t=-1,\ s=2$

Question 3. Consider the set

$$V = \{(x,\, y) \mid x \geq 0 \text{ and } y \geq 0\}$$

under the following operations:

$$(x_1,\, y_1) + (x_2,\, y_2) = (x_1 + x_2,\, y_1 y_2) \qquad k(x,\, y) = (kx,\, y)$$

a. *(2 marks)* Does V contain a zero vector? If so find it. Justify.

b. *(2 marks)* Does V contain the additive inverse (negative of the vector in the sense of a vector space) of $\vec{v} = (3,\, 2)$? If so find it. Justify.

c. *(1 mark)* Is V a vector space? Justify.

a) Let $\underline{v} = (x, y) \in V$ and $\underline{0} = (a, b)$

$$\underline{v} + \underline{0} = \underline{v}$$
$$(x,y) + (a,b) = (x,y)$$
$$(x+a,\, yb) = (x,y)$$

$x + a = x$ $\quad yb = y$

$a = 0$ $\quad \Rightarrow b = 1$

$\therefore \underline{0} = (0,1) \in V$

b) Let $\underline{w} = (x,y)$

$$\underline{v} + \underline{w} = \underline{0}$$
$$(3,2) + (x,y) = (0,1)$$
$$(3+x,\, 2y) = (0,1)$$

$3 + x = 0$ $\quad 2y = 1$

$x = -3$ $\quad y = \frac{1}{2}$

$\therefore \underline{w} = (-3, \frac{1}{2}) \notin V$ since $-3 \not\geq 0$.

c) Not a V.S. because not all vectors have additive inverses.

Question 4. *(5 marks)* Let $W = \{(f \mid f(-x) = -f(x)\}$. Determine whether W is a subspace of $V = \{f \mid f : \mathbb{R} \to \mathbb{R}\}$.

$W \neq \{\}$ since $0(x) = 0 \in W$ because $0(-x) = 0 = -0 = -0(x)$

① Closure under addition

Let $f, g \in W \Rightarrow f(-x) = -f(x)$

$g(-x) = -g(x)$

$f + g \in W$ since
$$(f+g)(-x) = f(-x) + g(-x)$$
$$= -f(x) - g(x)$$
$$= -(f(x) + g(x))$$
$$= -(f+g)(x)$$

② Closure under scalar multiplication

Let $f \in W \Rightarrow f(-x) = -f(x)$

$r \in \mathbb{R}$

$rf \in W$ since $(rf)(-x) = rf(-x) = r(-f(x)) = -rf(x) = -(rf)(x)$

$\therefore W$ is a subspace by the subspace test.

Bonus. *(3 marks)* Sketch $r(t) = (\sin t,\, \cos t,\, t)$ where $t \in \mathbb{R}$.