**Dawson College**: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2025: **Quiz 8** name: ____________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** *(5 marks)* Let $V$ be the solution space of the equation $4x - y + 2z = 0$, and let $W$ be the subspace of $\mathbb{R}^3$ spanned by $(1,\ 1,\ 1)$. Find a vector $\mathbf{v}$ in $V$ and a vector $\mathbf{w}$ in $W$ for which $\mathbf{v} + \mathbf{w} = (1,\ 0,\ 1)$

**Question 2.** *(5 marks)* If $\{\mathbf{v}_1,\ \ldots,\ \mathbf{v}_n\}$ are linearly dependent nonzero vectors, then at least one vector $\mathbf{v}_k$ is a unique linear combination of $\{\mathbf{v}_1,\ \ldots,\ \mathbf{v}_{k-1}\}$ .

**Question 3.** Let $W$ be the subspace of all polyomials in $\mathbb{P}_3$ such that $p(1) = 0$

a. *(4 marks)* Find a basis $\mathscr{B}$ of $W$.

b. *(1.5 marks)* State $\dim(\mathbb{P}_3)$, $\dim(W)$, and $\dim\left(\{0+0x+0x^2+0x^3\}\right)$.

c. *(1 mark)* Find the coordinate vector of $p(x) = 1+x+x^2-3x^3$ relative to the basis found in part a.

**Question 4.** *(3.5 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

There are only three distinct two-dimensional subspaces of $\mathbb{P}_2$.