Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2025: **Quiz 1** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. Consider a system of linear equations with augmented matrix A. If there is a unique solution then A has no row of zeros.

False, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ has solution $\left.\begin{matrix} x+0y=0 \\ 0x+y=0 \\ 0x+0y=0 \end{matrix}\right\} \Rightarrow \begin{matrix} x=0 \\ y=0 \end{matrix}$

$\therefore$ a unique solution $(x,y)=(0,0)$ and has a row of zeros.

b. If each equation in a consistent linear system is multiplied through by a constant c, then all solutions to the new system can be obtained by multiplying solutions from the original system by c.

False, the system $\begin{matrix} x+y=1 \\ y=1 \end{matrix}$ has solution $(x,y)=(0,1)$

but $\begin{matrix} cx+cy=c \\ cy=c \end{matrix}$ still has solution $(x,y)=(0,1)$ provided $c\neq 0$

Question 2. *(3 marks)* Find (if possible) conditions on a and b such that the system has no solution, one solution, and infinitely many solutions. Justify.

$$\begin{cases} ax & +y=1 \\ 2x & +y=b \end{cases} \Rightarrow \begin{cases} y=-ax+1 \\ y=-2x+b \end{cases}$$

For the system to have no solution, the two lines cannot have a point in common. So the lines need to be parallel with different y-int.

$\therefore$ $-a=-2$ and $b\neq 1$
$a=2$

For the system to have ∞-many solutions, the two lines need to be identical that is, same slope and y-intercept

$\therefore$ $-a=-2$ and $b=1$
$a=2$

For the system to have a unique sol. the two lines need to have different slopes.

$\therefore$ $-a\neq -2$
$a\neq 2$

Question 3. *(2 marks)* Consider the following augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 2 & 4 & 6 \end{bmatrix}$$

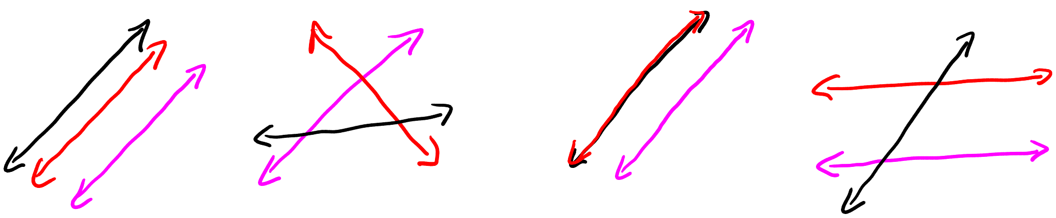
Find a row which can be added to the augmented matrix to make a new system with four equations which has no solutions. Justify.

The following equation $0x+0y=1$ is inconsistent

$\therefore$ The system whose augmented matrix is $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 2 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$

is inconsistent.

Question 4. *(2 marks)* Illustrate **all** relative positions of lines in a inconsistent linear system consisting of three lines.



Question 5. *(3 marks)* Find the solution set of the following equation $4x_1 - x_2 + x_3 + 5x_4 = 1$. Find a particular solution where $x_2 = 1$, $x_4 = -1$.

Let $x_2 = s$
$x_3 = t$
$x_4 = r$

$4x_1 - s + t + 5r = 1$

$x_1 = \frac{1}{4} + \frac{1}{4}s - \frac{1}{4}t - \frac{5}{4}r$

$\therefore$ $x_1 = \frac{1}{4} + \frac{1}{4}s - \frac{1}{4}t - \frac{5}{4}r$
$x_2 = s$
$x_3 = t$ $\qquad s,t,r \in \mathbb{R}$
$x_4 = r$

If $x_2 = 1 \Rightarrow s = 1$
$x_4 = -1 \Rightarrow r = -1$

and let $t = 0$

$\therefore$ $x_1 = \frac{1}{4} + \frac{1}{4}(1) - \frac{1}{4}(0) - \frac{5}{4}(-1)$
$= \frac{7}{4}$

$\therefore$ $(x_1, x_2, x_3, x_4) = (\frac{7}{4}, 1, 0, -1)$